中正嶺學報 第三十四卷 第二期 民國 95.5.
JOURNAL OF C.C.I.T., VOL.34, NO.2, May., 2006

Multiobjective State-Feedback Control for Stochastic Large-Scale System via LMI Approach

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ABSTRACT

This paper investigates a state-feedback control such that multiobjective performance requirements for stochastic large-scale systems can be achieved. The required objectives consist of disturbance attenuation, upper bound on state covariance, and regional constraint on pole placement. Based on the feasibility of a matching condition, three controller design approaches, the decoupled, centralized, and decentralized control, are addressed accordingly. By assuming suitable form of the quadratic Lyapunov function, the constructive conditions to deliver the desired multiobjective performance for the considered stochastic large-scale systems are derived in terms of linear matrix inequalities (LMIs). The effectiveness of the proposed methods is illustrated by a numerical example.

Keywords: LMIs, stochastic control, large-scale system, multiobjective constraint

線性矩陣不等式應用於隨機大型系統之多目標狀態回授控制器設計

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摘要

本文之目的係針對隨機大型系統，設計符合外擾衰減限制、狀態協方差上界限制及極點配置等多目標性能需求之狀態回授控制器。基於大型系統之匹配條件特性與利用適當的李亞普諾夫函數，本文採用線性矩陣不等式方法，分別針對解耦式、集中式及分散式等三種不同設計方式，推導其控制器存在之充分條件。最後，並以數值範例驗證本文所提出方法的實效性。

關鍵詞：線性矩陣不等式，隨機控制，大型系統，多目標性能限制

文稿收件日期 93.4.16; 文稿修正後接受日期 94.10.2.
Manuscript received April 16, 2004; revised October 2, 2005.

I. INTRODUCTION

Large-scale systems, consisting of a set of lower-dimension interconnected subsystems, are frequently encountered in the real world such as power systems, digital communications networks, flexible manufacturing networks, etc. [1]. Due to the existence of interconnections among subsystems, the controller design of a large-scale system is in general much more difficult than that of individual subsystems. Depending on the availability of the state information and the structural characteristics of the composed subsystems, the large-scale systems can be suitably handled by the approaches of decoupled, centralized, or decentralized control.

The decoupled control approach is based on the existence of certain matching condition such that the controller design of the large-scale system can be reduced to that of a set of subsystems without interconnection [2, 3]. In the case that the matching condition is not satisfied, we can utilize the approach of centralized control by incorporating each individual subsystem as an augmented overall system. The obtained controller then consists of individual and cross-connected state-feedback gains [4, 5]. However, it may be cost more and even impossible to design controller based on information such as state variables that need to be delivered from other subsystems. Alternatively, the scheme of decentralized control based only on local information has the merits of simplicity and applicability and therefore has received considerable attention in the literature [6-8].

In this paper, the mentioned types of design methodologies are investigated for large-scale systems with stochastic external input. In the resulted overall closed-loop systems, the individual subsystems are required to satisfy certain multiobjective performance. In general, a multiobjective control problem considers a mix of time- and frequency-domain specifications as presented in [9, 10] and references therein.

In the study of stochastic large-scale system, it is desired that the state variables of the closed-loop systems are maintained within certain level of root mean squared values. One way is to address the state covariance upper bound condition during controller design [11-14]. In the decoupled control scheme, based on the property of matching condition, generalized inverse and upper bound covariance control (UBCC) techniques are utilized to derive a state-feedback control law such that the closed-loop states of individual subsystems meet a given covariance upper bound condition and the regulated outputs related to the stochastic input satisfy some H_∞ performance level [2]. It was further explored in [15] such that additional control objective with disk region constraint on the closed-loop poles of individual subsystems can be achieved.

In case that the matching condition is not satisfied, the designs of [2, 15] are no longer applicable. Though we may proceed with the centralized control approach to deal with this controller design, it is desirable to introduce a simpler control law by studying decentralized control design for the considered stochastic large-scale systems.

Recently, the multiobjective control problems have been widely addressed in control engineering via linear matrix inequality (LMI) method [16-18] which can be solved by efficiently numerical algorithms [19, 20].

Specifically, the main purpose of this paper is to propose a decentralized control design by using the LMI method such that the considered stochastic large-scale systems can satisfy desired multiobjective performances consisting of H_∞-norm disturbance attenuation level, upper bound on individual state variance, and disk regional pole placement. Meanwhile, the problem has not been addressed in [2, 15] for the situation that the matching condition is not satisfied, which can be solved by the approach proposed in this paper. The control law takes the form of state feedback and the derivation is based on a composite quadratic Lyapunov function. The comparison between this approach and [2, 15] is also shown in the given examples.

The remainder of this paper is organized as follows. Section 2 presents the statement and formulation of the decoupled, centralized, and decentralized control approaches for considered stochastic large-scale systems with the desired multiobjective performance. Section 3 introduces the derivations of state-feedback control laws for the three typical types of control approaches such that the specified multiobjective performance can be achieved. Then, Section 4 presents a numerical example for illustration. Finally, Section 5 gives conclusion.

II. PROBLEM STATEMENT AND FORMULATION

Consider a stochastic large-scale system which consists of N interconnected linear time-invariant subsystems and can be described as

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t) \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij} x_j(t) + D_i w_i(t) \\ &\quad i = 1,\ 2, \cdots, N \end{aligned} \tag{1a}$$

$$y_i(t) = F_i x_i(t) \tag{1b}$$

where $x_i(t) \in R^{n_i}$, $u_i(t) \in R^{m_i}$ and $w_i(t) \in R^{m_i}$ are the state, control input, and noise input to the i^{th} subsystem, respectively. The dimensions of the system matrices are $A_i, A_{ij} \in R^{n_i \times n_i}$, $B_i, D_i \in R^{n_i \times m_i}$ and $F_i \in R^{m_i \times n_i}$, where A_{ij} for i, $j = 1, 2, \cdots, N$, $j \neq i$ are the interconnection matrices between the i^{th} and j^{th} subsystem. It is assumed that the matrix pairs $(A_i,\ B_i)$ are stabilizable, moreover, the zero-mean white noise $w_i(t)$, $E(w_i(t)) = 0,$ satisfies the following uncorrelated properties:

$$\begin{aligned} &E(w_i(t) x_i^T(t)) = 0, \\ &E(w_i(t) w_i^T(t)) = I_i, \\ &E(w_i(t) w_j^T(t)) = 0, \end{aligned} \tag{2}$$

for $i, j = 1,2,\ldots,N,\quad j \neq i.$

The purpose of this paper is to investigate suitable design approaches based on different system characteristics by using linear state-feedback information to achieve some desired multiobjective performance. In the following of this section, the various types of system structure and corresponding design approaches are introduced and formulated; then, the objective performance of interest is presented in sequence.

2.1 Decoupled Control Design Approach

The decoupled control method is based on the existence of matching condition [2] that the interconncection matrix A_{ij} and input matrix B_i are spanned by the same basis. Equivalently, the matching condition can be written as

$$rank\ [B_i, A_{ij}] = rank\ [B_i], \tag{3}$$

and it can be found in many practical large-scale mechanical systems [2]. Condition (3) enables the existence of a constant matrix E_j such that

$$A_{ij} = B_i E_j. \tag{4}$$

In this case, the given stochastic large-scale system (1) can be decoupled and reformulated as a set of linear individual closed-loop subsystem without any interconnection. Let the state-feedback control law for the i^{th} subsystem be denoted as

$$u_i(t) = G_i x_i(t) - \sum_{\substack{j=1 \\ j\neq i}}^{N} E_j x_j(t) \tag{5}$$

where matrix $E_j \in R^{m_j \times n_j}$ is determined by the matching conditions (3) and (4), and local state-feedback gain $G_i \in R^{m_i \times n_i}$ is to be determined using the subsequent controller construction algorithm. After substituting the control law (5) into (1), we have the following decoupled closed-loop system:

$$\begin{aligned}\dot{x}_i(t) &= A_i x_i(t) + B_i[G_i x_i(t) - \sum_{\substack{j=1 \\ j\neq i}}^{N} E_j x_j(t)] \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij} x_j(t) + D_i w_i(t) \\ &= (A_i + B_i G_i) x_i(t) + D_i w_i(t) \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N} (A_{ij} - B_i E_j) x_j(t) \\ &= \hat{A}_i x_i(t) + D_i w_i(t), \quad i = 1,2,\ldots,N\end{aligned} \tag{6a}$$

$$y_i(t) = F_i x_i(t), \tag{6b}$$

where $\hat{A}_i = A_i + B_i G_i$. It is noted that though only local state-feedback gain to be constructed, the state information from other subsystem, $x_j(t),\ j \neq i$, is still needed in the control law (5) to facilitate this decoupled control scheme.

2.2 Centralized Control Design Approach

In this subsection, we discuss the situation when the matching condition is not satisfied in system (1). We are looking for inter-subsystem state feedback gain G_{ij} for the i^{th} subsystem based on the state information of the j^{th} subsystem x_j, instead of the matching condition matrix E_j as shown in (5) for all subsystems $i = 1, 2, \cdots, N$, using the subsystem state x_j. That is, the state-feedback control law for the i^{th} subsystem can be written as

$$u_i(t) = G_i x_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} G_{ij} x_j(t). \tag{7}$$

The overall system (1) in the closed-loop form then can be expressed as

$$\dot{x}(t) = (A + BG)x(t) + Dw(t) \tag{8a}$$

$$= \hat{A}x(t) + Dw(t),$$

$$y(t) = Fx(t), \tag{8b}$$

where the integrated variables are

$$x(t) = \begin{bmatrix} x_1^T(t) & x_2^T(t) & \cdots & x_N^T(t) \end{bmatrix}^T,$$

$$y(t) = \begin{bmatrix} y_1^T(t) & y_2^T(t) & \cdots & y_N^T(t) \end{bmatrix}^T,$$

$$w(t) = \begin{bmatrix} w_1^T(t) & w_2^T(t) & \cdots & w_N^T(t) \end{bmatrix}^T.$$

The overall matrix parameters are, $F = diag\{F_i\}$, $D = diag\{D_i\}$, $B = diag\{B_i\}$, $i = 1, 2, \cdots, N$, and

$$A = \begin{bmatrix} A_1 & A_{12} & \cdots & A_{1N} \\ A_{21} & A_2 & \cdots & A_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1} & A_{N2} & \cdots & A_N \end{bmatrix}, \quad G = \begin{bmatrix} G_1 & G_{12} & \cdots & G_{1N} \\ G_{21} & G_2 & \cdots & G_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ G_{N1} & G_{N2} & \cdots & G_N \end{bmatrix}. \tag{9}$$

In comparison with the decoupled control approach, the off-diagonal blocks of state feedback matrix, G_{ij} are reduced to $G_{ij} = -E_j$, $i = 1, 2, \cdots, N$, in case that the matching conditions (3) and (4) are satisfied. For the general situation without concerning the matching condition, this centralized control approach needs the state information for all of the subsystems and can be denoted as

$$u(t) = Gx(t), \tag{10}$$

where $u(t) = \begin{bmatrix} u_1^T(t) & u_2^T(t) & \cdots & u_N^T(t) \end{bmatrix}^T$.

2.3. Decentralized Control Design Approach

In this approach, only local state information is assumed to be available for controller design for each subsystem, namely that the form of control law takes

$$u_i(t) = G_i x_i(t), \tag{11}$$

where neither the matching condition matrix E_j in the decoupled design nor the coupling state feedback matrix G_{ij} in the centralized design are assumed. In this design approach, the state feedback matrix for the overall system G as shown in (9) can be reduced to the diagonal form $G = diag\{G_i\}$ such that the decentralized information structure constraints [9] are satisfied. After substituting the control law (11) into (1), we have the following closed-loop system for the decentralized design:

$$\begin{aligned} \dot{x}_i(t) &= (A_i + B_i G_i)x_i(t) + D_i w_i(t) \\ &\quad + \sum_{\substack{j=1 \\ j \neq i}}^{N} A_{ij} x_j(t), \\ &= \hat{A}_i x_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{N} A_{ij} x_j(t) + D_i w_i(t), \end{aligned} \tag{12a}$$

$$y_i(t) = F_i x_i(t). \tag{12b}$$

The control objectives concerned in this paper include H_∞ performance on the disturbance attenuation, constraints on the state covariance upper bound, and specifications on the pole placement region. Each of them is formulated in the following.

Objective 1: Performance Level of Disturbance Attenuation

In the considered stochastic large-scale system (1), the effect of the disturbance input $w_i(t)$ on the plant output y_i should be kept small for each subsystem. Under the assumption that the i^{th} subsystem is controlled to be stable, let $H_i(s)$ denote the closed-loop transfer function from $w_i(t)$ to $y_i(t)$. The desired H_∞ performance level is described as [21]:

$$\|H_i(s)\|_\infty = \sup_{w_i(t)\neq 0} \frac{\|y_i(t)\|_2}{\|w_i(t)\|_2}$$
$$= \sup_{w_i(t)\neq 0} \left(\frac{\int_0^\infty y_i^T(t)y_i(t)dt}{\int_0^\infty w_i^T(t)w_i(t)dt}\right)^{1/2} \qquad (13)$$
$$= \sup_{w_i(t)\neq 0} \left(\frac{\int_0^\infty \|y_i(t)\|^2 dt}{\int_0^\infty \|w_i(t)\|^2 dt}\right)^{1/2} \le \gamma_i,$$

where the performance level upper bound γ_i can be implemented as a constraint to be met or a parameter to be minimized during the controller construction. The performance level (13) is designated for the approaches of decoupled control as well as decentralized control, in which the closed-loop systems are shown in (6) and (12), respectively. As for the case of centralized control, only the following overall performance of the closed-loop system (8) is considered:

$$\|H(s)\|_\infty = \sup_{w(t)\neq 0} \frac{\|y(t)\|_2}{\|w(t)\|_2} \qquad (14)$$
$$= \sup_{w(t)\neq 0} \left(\frac{\int_0^\infty y^T(t)y(t)dt}{\int_0^\infty w^T(t)w(t)dt}\right)^{1/2}$$
$$= \sup_{w(t)\neq 0} \left(\frac{\int_0^\infty \|y(t)\|^2 dt}{\int_0^\infty \|w(t)\|^2 dt}\right)^{1/2} \le \gamma.$$

Therefore, no exact information can be inferred regarding the performance level γ_i of individual subsystem.

Objective 2: Constraints on State Covariance Upper Bound

Besides the signal amplitude considered in the output channel, we are also interested in the state covariance induced by the external disturbance input. For the cases of decoupled and decentralized control, the state covariance of each individual i^{th} subsystem can be required to satisfy the following upper bound constraints:

$$\left[\tilde{X}_i\right]_{kk} \le [X_i]_{kk} = Var(x_{ik}(t)) \le (\sigma_k^2)_i, \quad (15)$$

where $k = 1, 2, \cdots, n_i$, and $(\sigma_k)_i$ denote the constrains of the root-mean-square (RMS) values of the k^{th} state variable; $[X_i]_{kk}$ denote the k^{th} diagonal element of state covariance upper bound matrix X_i ; $Var(x_{ik}(t))$ and $[\tilde{X}_i]_{kk}$ denote the k^{th} diagonal element of the state covariance matrix $\tilde{X}_i$ which is defined as

$$\tilde{X}_i = \lim_{t\to\infty} E(x_i(t)x_i^T(t)). \qquad (16)$$

As for the case of centralized control, the constraints on state covariance upper bound are expressed in terms of the overall large-scale system:

$$\left[\tilde{X}\right]_{kk} = Var(x_k(t)) \le [X]_{kk} = (\sigma_k^2), \qquad (17)$$

$$k = 1,2,\ldots,\sum_{i=1}^{N} n_i,$$

where each design parameter is denoted according to its appearance in the content of overall system formulation.

Objective 3: Constraints on Pole Placement Region

The issue of transient response of the designed closed-loop system is addressed by properly specifying the locations of its poles. In this paper for the considered stochastic large-scale system, the region of interest in the complex z-plane is described by the LMI condition [18]:

$$D = \{z \in C : f_D(z) = L + zM + \bar{z}M^T < 0\}, \quad (18)$$

where C denotes the set of complex number; $L = L^T$ and M are real matrix parameters for choosing a suitable convex region by defining the characteristic function $f_D(z)$. Specifically, we consider the region of the disk $D(-q_i, \rho_i)$ with center at $(-q_i, 0)$ and radius $0 < \rho_i < q_i$ for the closed-loop pole of the i^{th} subsystem. Indeed, the disk region $D(-q_i, p_i)$ in the complex plane $z = \sigma + j\omega$, can be described as

$$(q_i + \sigma)^2 + \omega^2 = (q_i + z)(q_i + \bar{z}) < \rho_i^2. \quad (19)$$

By the property of Schur's complement [16], we have the characteristic function $f_D(z)$ of the disk region $D(-q_i, \rho_i)$ from (19) as follows:

$$f_D(z) = \begin{bmatrix} -\rho_i & q_i + z \\ q_i + \bar{z} & -\rho_i \end{bmatrix} < 0. \quad (20)$$

In comparison with the defined LMI condition (18), the matrix parameters for the disk region $D(-q_i, \rho_i)$ are

$$L = \begin{bmatrix} -\rho_i & q_i \\ q_i & -\rho_i \end{bmatrix}, \quad M = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}. \quad (21)$$

The considered i^{th} subsystem is called D-stable if the eigenvalues of the closed-loop system matrix $\hat{A}_i$ from (6) for decoupled control or (12) for decentralized control lie within the disk region $D(-q_i, \rho_i)$, i.e.

$$\lambda(\hat{A}_i) \in D(-q_i, \rho_i). \quad (22)$$

As for the case of centralized control, the closed-loop poles are specified in terms of the overall system matrix A from (8) and required to lie within the disk region $D(-q, \rho)$ with suitable chosen parameters $q < \rho < 0$.

III. DESIGN of MULTI OBJECTIVE CONTROL LAWS

In this paper, state-feedback control design is conducted for the stochastic large-scale system (1) in the formulation of the aforementioned decoupled, centralized, and decentralized control approaches with their closed-loop systems described in (6), (8), and (12), respectively. Specifically, only the approaches of decoupled and decentralized control will be discussed in detail since the formulated closed-loop system of centralized control take the same expression as the decoupled control except that a notation with respect to the integrated overall large-scale system is used. In the following derivation, a suitable quadratic Lyapunov function is assumed for constructing the desired multiobjective performance in terms of the LMI conditions.

3.1 Performance Level of Disturbance Attenuation

3.1.1 Decoupled Control Approach

In considering the performance related to both the amplitude attenuation level in the output channel and the state covariance upper bound in the presence of stochastic external input, we resort to the result for using a Riccati equation to address the LQG control design with a H_∞ performance bound, which was proposed for a single system design in [12] and quoted for the case of large-scale system in [2]. It is restated as the following lemma for the considered decoupled control approach with the closed-loop system (6).

Lemma 3.1. *[2] Consider the stochastic large-scale system (1) with a matching condition matrix E_j such that (4) is satisfied. In the decoupled control formulation with control input (5), the closed-loop representation of (1) is written as (6). Under the assumption that the matrix pairs (A_i, B_i) are stabilizable, if there exist state-feedback gain G_i and positive symmetric matrix X_i such that*

$$\begin{aligned}&(A_i + B_i G_i)X_i + X_i(A_i + B_i G_i)^T \\ &+ \tilde{\gamma}_i^{-2} X_i F_i^T F_i X_i + D_i D_i^T = 0,\end{aligned} \tag{23}$$

then the closed-loop transfer function of (6) satisfies the H_∞-norm condition as defined in (13):

$$\|H_i(s)\|_\infty \le \tilde{\gamma}_i. \tag{24}$$

Moreover, the matrix X_i is also the upper bound of the state covariance matrix X_i as defined in (16), that is,

$$\tilde{X}_i \le X_i. \tag{25}$$

Indeed, for a positive number γ_i such that $\gamma_i > \tilde{\gamma}_i$, the following matrix inequality is true for the state-feedback gain G_i and positive matrix X_i obtained from (23):

$$\begin{aligned}&(A_i + B_i G_i)X_i + X_i(A_i + B_i G_i)^T \\ &+ \gamma_i^{-2} X_i F_i^T F_i X_i + D_i D_i^T < 0,\end{aligned} \tag{26}$$

and we have

$$\|H_i(s)\|_\infty < \gamma_i, \tag{27}$$

as well as the state covariance upper bound condition (25). Therefore, the statement that the condition (23) holds for $G_i, X_i, \tilde{\gamma}_i$ to achieve performance (24) and (25) is equivalent to that the condition (26) holds for G_i, X_i, γ_i to achieve performance (25), and (27). Hence, we concluded that the condition (26) is equivalent to (23) with $\tilde{\gamma}_i$ replaced by γ_i. Meanwhile, the matrix inequality condition (26) provides an alternative way to construct state-feedback gain such that both the performance regarding disturbance attenuation and state covariance can be satisfied. However, since the matrix inequality (26) defined nonlinearly in the matrix variables G_i and X_i, its numerical solution does not possess the merit of convex property and therefore the resulted solution may be only local optimal. This desired convexity can be fixed as follows. By using Schur's Complement and denoting $L_i = G_i X_i$, we can rewrite (26) as

$$\begin{bmatrix} A_i X_i + B_i L_i + X_i A_i^T \\ + L_i^T B_i + D_i D_i^T & X_i F_i^T \\ F_i X_i & -\gamma_i^2 I \end{bmatrix} < 0, \tag{28}$$

which is already linear in the matrix variables L_i and X_i. Once the numerical solution for L_i and X_i is obtained, the original state-feedback gain G_i can be reconstructed as $G_i = L_i X_i^{-1}$. We then have the following proposition to address the design performance for both the external disturbance attenuation and related state covariance.

Proposition 3.1. *In the formulation of decoupled control, (6) is the closed-loop representation of the stochastic large-scale system (1) with the control input (5). If there exist matrix L_i and positive symmetric matrix X_i such that the LMI condition (28) holds, then the closed-loop transfer function of (6) satisfies the H_∞-norm condition (27) and the state covariance X_i satisfies the upper bound condition (25). Moreover, one feasible state-feedback gain is $G_i = L_i X_i^{-1}$.*

On the other hand, we define the quadratic Lyapunov function as

$$\begin{aligned} &V(x_i(t)) = x_i^T(t) P_i x_i(t), \\ &P_i = P_i^T > 0, \end{aligned} \tag{29}$$

according to $It\hat{o}'s$ differential rule [22] the time derivative of the quadratic Lyapunov function is,

$$\begin{aligned} \frac{d}{dt} V(x_i(t)) &= x_i^T P_i (A_i + B_i G_i) x_i \\ &+ x_i^T (A_i + B_i G_i)^T P_i x_i + tr\{D_i^T P_i D_i\}. \end{aligned} \tag{30}$$

Suppose that $V(x_i(t)) = x_i^T(t) P_i x_i(t)$ satisfies

$$\begin{aligned} &\frac{d}{dt} V(x_i(t)) + y_i^T(t) y_i(t) \\ &\quad - \gamma_i^2 w_i^T(t) w_i(t) < 0, \end{aligned} \tag{31}$$

exists for all $x_i(t)$, $y_i(t)$, and $w_i(t)$. Integrating the both side, we have the H_∞ norm disturbance attenuation performance

$$\begin{aligned} &\int_0^\infty \{\gamma_i^2 w_i^T(t) w_i(t) - y_i^T(t) y_i(t)\} dt \\ &= \gamma_i^2 \|w_i(t)\|_2^2 - \|y_i(t)\|_2^2 > V(x_i(t)) \ge 0. \end{aligned} \tag{32}$$

Therefore, the quadratic stability as well as the disturbance attenuation with a performance level γ_i are established through (31). By substituting the closed-loop dynamics (6) into (31), and take expectation to both side of (31), we then have

$$E\left[\begin{bmatrix} x_i(t) \\ w_i(t) \end{bmatrix}^T \begin{bmatrix} (A_i + B_i G_i)^T P_i \\ + P_i(A_i + B_i G_i) & P_i D_i \\ + F_i^T F_i & \\ D_i^T P_i & -\gamma_i^2 I_i \end{bmatrix} \begin{bmatrix} x_i(t) \\ w_i(t) \end{bmatrix}\right] < 0, \tag{33}$$

where $E[\cdot]$ is expectation operator. By Schur's complement, the inequality condition (33) is equivalent to

$$\begin{aligned}&(A_i + B_i G_i)^T P_i + P_i(A_i + B_i G_i) \\ &+ F_i^T F_i + \gamma_i^{-2} P_i D_i D_i^T P_i < 0.\end{aligned} \tag{34}$$

Pre- and post-multiplying both sides of (34) by $Q_i := P_i^{-1}$, we have

$$\begin{aligned}&(A_i + B_i G_i)Q_i + Q_i(A_i + B_i G_i)^T \\ &+ Q_i F_i^T F_i Q_i + \gamma_i^{-2} D_i D_i^T < 0.\end{aligned} \tag{35}$$

In the comparison with the state covariance upper bound X_i in (25) and the matrix inequality condition in (26), there exist matrices G_i, X_i such that the condition (26) holds if and only if the matrix variables G_i, Q_i with $Q_i = \gamma_i^{-2} X_i$ satisfy (35). Then, The corresponding state covariance upper bound condition becomes

$$\widetilde{X}_i \le \gamma_i^2 Q_i. \tag{36}$$

The inequalities (35) and (36) establish the desired performance of disturbance attenuation and state covariance upper bound by the derivation using a quadratic Lyapunov function. This result is stated as the following proposition with an application of Schur's complement to (35) and the use of denotation $G_i Q_i = \Gamma_i$.

Proposition 3.2. *In the formulation of decoupled control, (6) is the the closed-loop representation of the stochastic large-scale system (1) with the control input (5). If there exist Γ_i and positive symmetric matrix Q_i such that the following LMI condition holds:*

$$\begin{bmatrix} A_i Q_i + B_i \Gamma_i + Q_i A_i^T \\ + \Gamma_i^T B_i^T + \gamma_i^{-2} D_i D_i^T & Q_i F_i^T \\ F_i Q_i & -I \end{bmatrix} < 0, \tag{37}$$

then the closed-loop transfer function of (6) satisfies the H_∞-norm condition (27) and the state covariance X_i satisfies the upper bound condition (36). Moreover, one feasible state feedback gain is $G_i = \Gamma_i Q_i^{-1}$.

3.1.2 Decentralized Control Approach

In aspect of the decentralized control approach for the stochastic large-scale system (1), the resulted closed-loop dynamics is represented in (12), where the interconnected dynamics from other subsystems $\sum_{j=1, j\neq i}^{N} A_{ij} x_j(t)$, for $j \neq i$, still appear in the i^{th} subsystem dynamics. Therefore, neither the Lyapunov equation (23) in Lemma 3.1 nor the related LMI condition (28) in Proposition 3.1 can be applied directly to address the performance of disturbance attenuation level as well as state covariance upper bound under this decentralized control scheme. However, based on the derivation in Proposition 3.2 by assuming a quadratic Lyapunov function to ensure the existence of the inequality (31), we will be able to show suitable LMI condition such that both the performance of disturbance attenuation and state covariance upper bound can be established for the considered decentralized control design as the conditions (36) and (37) described in Proposition 3.2 for the case of decoupled control.

We shall now investigate the closed-loop dynamics (12) in the decentralized control scheme. Define the composite Lyapunov function as follows:

$$V(x(t)) = \sum_{i=1}^{N} V(x_i(t)) = \sum_{i=1}^{N} x_i^T(t) P_i x_i(t), \quad (38)$$

with $P_i = P_i^T > 0,$ to deal with the desired stability as well as the discussed performance of disturbance attenuation and state covariance upper bound in the aspect of the overall closed-loop system. According to $It\hat{o}'s$ formulations, the time derivative of (38) is,

$$\begin{aligned} \frac{d}{dt}V(x(t)) &= \sum_{i=1}^{N} \{x_i^T P_i (A_i + B_i G_i) x_i \\ &+ x_i^T (A_i + B_i G_i)^T P_i x_i + tr(w_i^T D_i^T P_i D_i w_i) \\ &+ \sum_{\substack{j=1 \\ j \neq i}}^{N} x_i^T(t)[P_i A_{ij} + A_{ij}^T P_i] x_j(t)\}. \end{aligned} \quad (39)$$

Similar to the statement as for as (31), if we can establish the condition

$$\begin{aligned} &\frac{d}{dt}V(x(t)) \\ &+ \sum_{i=1}^{N} \{y_i^T(t) y_i(t) - \gamma_i^2 w_i^T(t) w_i(t)\} < 0. \end{aligned} \quad (40)$$

Then, we obtain the H_∞-norm disturbance attenuation performance

$$\sum_{i=1}^{N} \int_0^\infty (\gamma_i^2 \|w_i(t)\|^2 - \|y_i(t)\|^2) dt > V(x(t)) \geq 0. \quad (41)$$

Because of the uncorrelated property of the white noise $w_i(t)$ as shown in (2), the condition of (41) builds the individual H_∞-norm performance

$$\int_0^\infty \|y_i(t)\|^2 dt - \int_0^\infty \gamma_i^2 \|w_i(t)\|^2 dt < 0. \quad (42)$$

Taking expectation for both sides of (40), we have

$$\begin{aligned} &E[\sum_{i=1}^{N} \{ \begin{bmatrix} x_i(t) \\ w_i(t) \end{bmatrix}^T \begin{bmatrix} (A_i + B_i G_i)^T P_i + P_i (A_i + B_i G_i) + F_i^T F_i & P_i D_i \\ D_i^T P_i & -\gamma_i^2 I_i \end{bmatrix} \begin{bmatrix} x_i(t) \\ w_i(t) \end{bmatrix} \\ &+ \sum_{\substack{j=1 \\ j \neq i}}^{N} x_i^T(t)[P_i A_{ij} + A_{ij}^T P_i] x_j(t)\}] < 0 \end{aligned} \quad (43)$$

where in the left-hand side, the first term is the same as the condition (33) for the case of decoupled control and the second term is provided by the interconnected dynamics among different subsystems. By the fact that for any scalar $\varepsilon_{ij} > 0,$ and any symmetric matrix variable $R_{ij} = R_{ij}^T > 0,$ the following expression (44) is always true.

$$\begin{aligned} &(\varepsilon_{ij}^{1/2} x_i^T(t) P_i A_{ij} R_{ij}^{1/2} - \varepsilon_{ij}^{1/2} x_j^T(t) R_{ij}^{-1/2}) \times \\ &(\varepsilon_{ij}^{1/2} x_i^T(t) P_i A_{ij} R_{ij}^{1/2} - \varepsilon_{ij}^{-1/2} x_j^T(t) R_{ij}^{-1/2})^T \geq 0. \end{aligned} \quad (44)$$

Then, we have

$$
\begin{aligned}
& x_i^T(t)[P_i A_{ij} + A_{ij}^T P_i]x_j(t) \\
& \le \varepsilon_{ij} x_i^T(t) P_i A_{ij} R_{ij} A_{ij}^T P_i x_i(t) \\
& \quad + \varepsilon_{ij}^{-1} x_j^T(t) R_{ij}^{-1} x_j(t).
\end{aligned} \tag{45}
$$

By substituting (45) into (43), we can obtain the equation as follows.

$$
\begin{aligned}
& E[\sum_{i=1}^{N}\{\begin{bmatrix} x_i(t) \\ w_i(t) \end{bmatrix}^T \begin{bmatrix} (A_i + B_i G_i)^T P_i + P_i(A_i + B_i G_i) + F_i^T F_i & P_i D_i \\ D_i^T P_i & -\gamma_i^2 I_i \end{bmatrix} \begin{bmatrix} x_i(t) \\ w_i(t) \end{bmatrix} \\
& + \sum_{\substack{j=1 \\ j\ne i}}^{N} x_i^T(t)[P_i A_{ij} + A_{ij}^T P_i]x_j(t)\}] \\
& \le E[\sum_{i=1}^{N}\{\begin{bmatrix} x_i(t) \\ w_i(t) \end{bmatrix}^T \begin{bmatrix} \Omega_i & P_i D_i \\ D_i^T P_i & -\gamma_i^2 I_i \end{bmatrix} \begin{bmatrix} x_i(t) \\ w_i(t) \end{bmatrix}\}] < 0,
\end{aligned} \tag{46}
$$

where

$$
\begin{aligned}
\Omega_i &= (A_i + B_i G_i)^T P_i + P_i(A_i + B_i G_i) \\
& \quad + F_i^T F_i + P_i \overline{A}_{ir} \prod_{ir} \overline{A}_{ir}^T P_i + \prod_{li}^{-1},
\end{aligned} \tag{47}
$$

$$
\begin{aligned}
& \overline{A}_{ir} = row\{A_{ij}\},\ \prod_{ir} = diag\{\varepsilon_{ij} R_{ij}\}, \\
& \prod_{li}^{-1} = diag\{\varepsilon_{ji}^{-1} R_{ji}^{-1}\}, \\
& j = 1, 2, \ldots, N,\ j \ne i.
\end{aligned} \tag{48}
$$

It follows the Schur's complement method [16], the equation (46) can be shown as the following equivalent condition.

$$
\begin{bmatrix} J_1 & 0 & \cdots & 0 \\ 0 & J_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & J_N \end{bmatrix} < 0 \tag{49}
$$

with

$J_i = \Omega_i + \gamma_i^{-2} P_i D_i D_i^T P_i,\ i = 1, 2, \cdots, N.$

Therefore, the overall closed-loop stability as well as the every individual H_∞-norm can be established if the following conditions are achieved simultaneously.

$$
J_i = \Omega_i + \gamma_i^{-2} P_i D_i D_i^T P_i < 0, \quad i = 1, 2, \cdots, N. \tag{50}
$$

Pre- and post-multiplying both sides of (50) by $Q_i := p_i^{-1}$ to yield

$$
\begin{aligned}
& Q_i(A_i + B_i G_i)^T + (A_i + B_i G_i)Q_i \\
& + Q_i F_i^T F_i Q_i + \overline{A}_{ir} \prod_{ir} \overline{A}_{ir}^T \\
& + \overline{Q}_i \prod_{li}^{-1} \overline{Q}_i^T + \gamma_i^{-2} D_i D_i^T < 0,
\end{aligned} \tag{51}
$$

where

$$
\overline{Q}_i = [Q_i, \ldots, Q_i]_{(n_i \times (N-1)n_i)}. \tag{52}
$$

By applying Schur's Complement to (51) and the use of denotation $G_i Q_i = \Gamma_i$, the above derivation for decentralized control is summarized as the following proposition.

Proposition 3.3 *In the formulation of decentralized control, (12) is the closed-loop representation of the stochastic large-scale system (1) with the control (11). If for every individual i^{th} subsystem, there exist Γ_i, positive symmetric matrices Q_i, and $\varepsilon_{ij} > 0$, $R_{ij} = R_{ij}^T > 0$, $j = 1,2,\cdots,N,\ j \ne i$ such that the following LMI conditions hold for $i = 1, 2, \cdots, N$, then each of the closed-loop transfer function of (12) satisfies the H_∞-norm condition (27) and the state covariance X_i satisfies the upper bound condition (36). Furthermore, the feasible state-feedback gain is $G_i = \Gamma_i Q_i^{-1}$.*

$$\begin{bmatrix} A_iQ_i + B_i\Gamma_i + Q_iA_i^T + \Gamma_i^TB_i^T \\ + \overline{A}_{ir}\prod_{ir}\overline{A}_{ir}^T + \gamma_i^{-2}D_iD_i^T & Q_iF_i^T & \overline{Q_i} \\ F_iQ_i & -I & 0 \\ \overline{Q_i} & 0 & -\prod_{li} \end{bmatrix} < 0. \tag{53}$$

3.2 Constraints on State Covariance Upper Bound

Consider the desired upper bound constraints on the state covariance as described in (15), which are specified in terms of the given upper bound of the RMS values of each state of individual i^{th} subsystem, $(\sigma_k^2)_i$, $k = 1, 2, \cdots, .n_i$. By the approach in Lemma 3.1 and Proposition 3.1, we have the upper bound condition for the state covariance matrix in (25), $\widetilde{X}_i \leq X_i$. Therefore, if given the magnitude constraints $(\sigma_k^2)_i \leq (\hat{\sigma}_k^2)_i$, we need the diagonal elements of the positive symmetry matrix X_i satisfying $[X_i]_{kk} = (\sigma_k^2)_i \leq (\hat{\sigma}_k^2)_i$, which can be implemented by appending the following inequality condition to the matrix variable X_i:

$$(\hat{\sigma}_k^2)_i - l_{ik}X_il_{ik}^T \geq 0, \tag{54}$$

where the row vector $l_{ik} \in R^{1\times n_i}$ with the k^{th} element equal to 1 and others equal to zero.

Alternatively, if we choose the algorithm in Proposition 3.2 for the design of decoupled control or Proposition 3.3 for the design of decentralized control, the upper bound condition for the state covariance matrix is given in (36), $\widetilde{X}_i \leq \gamma_i^2 Q_i$ and therefore we need $\gamma_i^2[Q_{kk}]_i \leq (\sigma_k^2)_i$. Similarly, it can be specified by adding the extra condition to the matrix variable Q_i:

$$(\sigma_k^2)_i - \gamma_i^2 l_{ik}Q_il_{ik}^T \geq 0. \tag{55}$$

3.3 Constraints on Pole Placement Region

We consider the closed-loop representations of the decoupled control in (6) and the decentralized control in (12), where the state dynamics matrix A_i is formed by $A_i = A_i + B_iG_i$. Though the closed-loop stability as well as the external stochastic input related output signal attenuation and state covariance are addressed as mentioned in Proposition 3.2 for decoupled control and Proposition 3.3 for decentralized control, it is desirable to impose suitable regional constraint on the closed-loop poles such that the performance of transient response can be specified.

The concerned disk region $D(-q_i, p_i)$ in the complex z-plane is designated by the LMI region with the variables $\sigma = \mathrm{Re}(z)$ and $\omega = \mathrm{Im}(z)$:

$$f_D(z) = L + zM + \bar{z}M^T < 0, \tag{56}$$

where the matrix parameters associated with $D(-q_i, p_i)$ are denoted in (21). In terms of the closed-loop state dynamics matrix A_i and positive symmetric matrix Q_i resulted from Proposition 3.2 or Proposition 3.3, the feasibility of pole region (56) is equal to the matrix inequality

$$R_D(\hat{A}_i, Q_i) = L \otimes Q_i + M \otimes (\hat{A}_i Q_i) + M^T \otimes (\hat{A}_i Q_i)^T, \tag{57}$$

which was proven in [17] and as a counterpart of Gutman's theorem for LMI regions. Since the expressions of $R_D(\hat{A}_i, Q_i)$ in (57) and $f_D(z)$ in (56) are related by the substitution $(Q_i, \hat{A}_i Q_i, Q_i \hat{A}_i^T) \leftrightarrow (1, z, \bar{z})$, the matrix inequality condition for the disk region $D(-q_i, p_i)$ as shown in (20) can be written as

$$\begin{bmatrix} -\rho_i Q_i & q_i Q_i + \hat{A}_i Q_i \\ q_i Q_i + Q_i \hat{A}_i^T & -\rho_i Q_i \end{bmatrix} < 0. \tag{58}$$

By the substitution of $\hat{A}_i = A_i + B_i G_i$ and $G_i Q_i = \Gamma_i$, we have the following LMI condition in terms of the matrix variables Q_i and Γ_i for addressing the disk region constraints of the closed-loop poles:

$$\begin{bmatrix} -\rho_i Q_i & q_i Q_i + A_i Q_i + B_i \Gamma_i \\ q_i Q_i + Q_i A_i^T + \Gamma_i^T B_i^T & -\rho_i Q_i \end{bmatrix} < 0, \tag{59}$$

where the resulted state-feedback gain $G_i = \Gamma_i Q_i^{-1}$ is the same as that stated in Proposition 3.2 for the decoupled control design or Proposition 3.3 for the case of decentralized control.

IV. NUMERICAL EXAMPLE

In this section, a numerical example is presented to demonstrate the effectiveness of the proposed multiobjective control approach for stochastic large-scale system under decoupled as well as decentralized control schemes.

We consider the following stochastic system consisting of two subsystems:

1^{st} *subsystem*:

$$\dot{x}_1(t) = A_1 x_1(t) + B_1 u_1(t) + A_{12} x_2(t) + D_1 w_1(t) \tag{60a}$$

$$y_1(t) = F_1 x_1(t), \tag{60b}$$

2^{nd} *subsystem*:

$$\dot{x}_2(t) = A_2 x_2(t) + B_2 u_2(t) + A_{21} x_1(t) + D_2 w_2(t) \tag{60c}$$

$$y_2(t) = F_2 x_2(t), \tag{60d}$$

where the states $x_1(t) = [x_{11} \quad x_{12}]^T$, $x_2 = [x_{21} \quad x_{22}]^T$ and the system matrices $A_1 = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $A_{12} = \begin{bmatrix} 0 & \delta_1 \\ 0.2 & 1 \end{bmatrix}$, $D_1 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}$, $F_1 = [1 \quad 0]$, $A_2 = \begin{bmatrix} 0 & 1 \\ -4 & 5 \end{bmatrix}$, $B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $A_{21} = \begin{bmatrix} 1 & 0.3 \\ \delta_2 & 0 \end{bmatrix}$, $D_2 = \begin{bmatrix} 0.25 \\ 0.5 \end{bmatrix}$, $F_2 = [1 \quad 0]$. Note that the system matrices A_1 and A_2 of the two subsystems possess positive eigenvalues $\{1, 2\}$ and $\{1, 4\}$, respectively. Parameters δ_1 and δ_2 determine the feasibility of the matching condition shown in (3), and (4). The regarding designs via decoupled and decentralized control approach will be demonstrated according to different values of δ_1 and δ_2.

The desired control objectives according to (13), (15), and (22) are specified as follows:

- Disturbance attenuation performance,

$$\|H_1(s)\|_\infty \le \gamma_1 = 1,$$
$$\|H_2(s)\|_\infty \le \gamma_2 = 0.8. \tag{61}$$

• State covariance upper bound constraints,

$$\begin{aligned} Var(x_{11}(t)) &\le (\sigma_1^2)_1 = 0.25, \\ Var(x_{12}(t)) &\le (\sigma_2^2)_1 = 1.5, \\ Var(x_{21}(t)) &\le (\sigma_1^2)_2 = 0.5, \\ Var(x_{22}(t)) &\le (\sigma_2^2)_2 = 2. \end{aligned} \tag{62}$$

• Pole placement region constraints,

$$\begin{aligned} \lambda(\hat{A}_1) &\in D(-q_1, \rho_1) = D(-25, 22), \\ \lambda(\hat{A}_2) &\in D(-q_2, \rho_2) = D(-20, 18). \end{aligned} \tag{63}$$

4.1 Decoupled Control Design

In the case that the parameters in the interconnection matrices A_{12}, and A_{21} are $\delta_1 = 0$, and $\delta_2 = 0$ such that the matching condition shown in (3) is satisfied, then the decoupled control approach for considered multiobjective design can be proceeded as follows:

1. The matching condition matrices E_1, and E_2 that satisfy $A_{21} = B_2 E_1$, $A_{12} = B_1 E_2$ as mentioned in (4) can be constructed as $E_1 = [0.2, 1]$, $E_1 = [1, 0.3]$. Then, under the structure of the state-feedback control law as given in (5), the considered system (60) is decoupled and formulated in the form as shown in (6).
2. The state-feedback gain matrices G_1, and G_2 in the control law that deliver the above design specifications (61), (62), and (63) for the closed-loop system of (60) can be constructed through the LMI conditions (37), (55), and (59) as,

$$\begin{aligned} G_1 &= [-73.6087, -26.4319], \\ G_2 &= [-32.4173, 68.0944]. \end{aligned} \tag{64}$$

3. The complete control laws for each subsystem are then,

$$u_1(t) = [-73.6087, -26.4319]x_1(t) - [0.2, 1]x_2(t),$$
$$u_2(t) = [-32.4173, 68.0944]x_2(t) - [1, 0.3]x_1(t).$$

By substituting the control law (64) into (60), the closed-loop poles of two subsystems are found to be $\lambda(\hat{A}_1) = \{-3.8683, -19.5456\}$, and $\lambda(\hat{A}_2) = \{-5.1275, -22.2898\}$, that all lie within the specified LMI region shown in (63). In the simulation, the frequency responses of each subsystem as shown in Figs 1, and 2 demonstrate that the H_∞-norm performance specifications given in (61) are satisfied, where the dotted lines ".." denote the designed upper bounds and solid lines denote the actual frequency responses. With the assumed initial states, $x_1(0) = [4, 7]^T$, $x_2(0) = [5, 8]^T$, the time responses of each subsystem are shown in Figs 3 and 4, where the dotted lines denote the zero mean, unitary variance noise input sequences $w_1(t)$, $w_2(t)$ generated by the MATLAB randn command, and the solid lines denote the actual time responses.

4.2 Decentralized Control Design

In case that the parameters in the interconnection matrices A_{12}, and A_{21} are assumed to be $\delta_1 = 0.1$, and $\delta_2 = 0.2$, then the matching condition shown in (3) is not satisfied and the introduced decoupled control approach is no longer applicable. By using the class of decentralized control law as shown in (11), in which only the state information of local subsystem is needed, the closed-loop system of (60) is written in the form of (12).

In terms of the LMI conditions (53), (55), and (59), the state-feedback gain matrices G_1, and G_2 can be constructed such that the design specifications (61), (62), and (63) could be satisfied. Note that in the implementation of (53), the diagonal block matrices Π_{ir}, Π_{li} for the i^{th} subsystem shown in (48) contain the items of variable multiplication $\varepsilon_{ij}R_{ij}$, $\varepsilon_{ji}R_{ji}$, $j=1,2,\cdots,N,\ j\neq i$. Without introducing any conservatism, we can simply pick values for the scalar parameters $\varepsilon_{ij}=\varepsilon_{ji}=1$. The resulting decentralized state-feedback control laws are

$$\begin{aligned} u_1(t) &= [-134.21111, -43.1782]x_1(t), \\ u_2(t) &= [-34.9891, 75.3594]x_2(t). \end{aligned} \tag{65}$$

By substituting the control laws (65) into (60), the individual closed-loop poles of each subsystem are indeed located inside the specified LMI region (63), respectively,

$$\begin{aligned} \lambda(\hat{A}_1) &= \{-3.7379, -36.4402\}, \\ \lambda(\hat{A}_2) &= \{-5.2814, -24.7077\}. \end{aligned} \tag{66}$$

However, in this decentralized control scheme, the interconnected items between subsystems $A_{ij}x_j$, $j=1,2,\cdots,N,\ j\neq i$, appeared in the overall closed-loop system (12) need to be taken into account. As in the derivation of Subsection 3.1.2, the overall closed-loop stability of (12) as well as the individual H_∞-norm performance (27) and state covariance upper bound condition (36) are established by using the composite Lyapunov function (38) and summarized in Proposition 3.3.

By expressing the overall closed-loop system (12) in the centralized control formulation (8) with state feedback matrix G in (9) replaced by $G=diag\{G_i\}$, $i=1,2,\cdots,N$ the closed-loop poles are found to be $\lambda(\hat{A})=\{-3.7061, -5.3188, -24.9605, -36.4519\}$ which are quite close to $\{\lambda(\hat{A}_1), \lambda(\hat{A}_2)\}$ and even the original pole placement specifications in terms of individual subsystem (63) are still satisfied for the overall system.

In the simulation based on the centralized overall formulation, the frequency responses in Figs 5 and 6 show that the H_∞-norm performance specifications (61) can be satisfied by each individual subsystem. As for the time response simulation with the same conditions as given in the case of decoupled design, Figs 7 and 8 show that satisfactory performance is obtained as well.

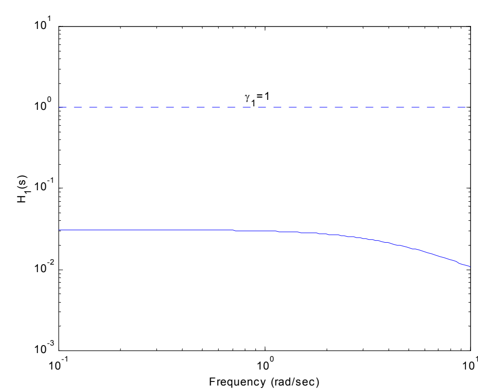

Fig. 1. The frequency response of the first subsystem in decoupled control design.

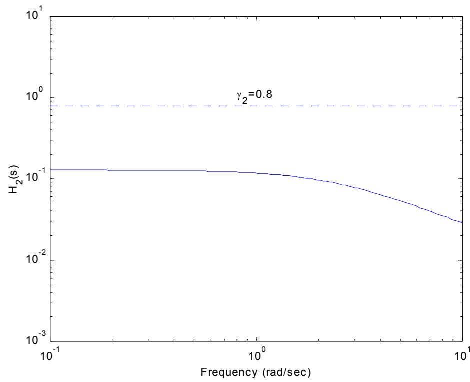

Fig. 2. The frequency response of the second subsystem in decoupled control design.

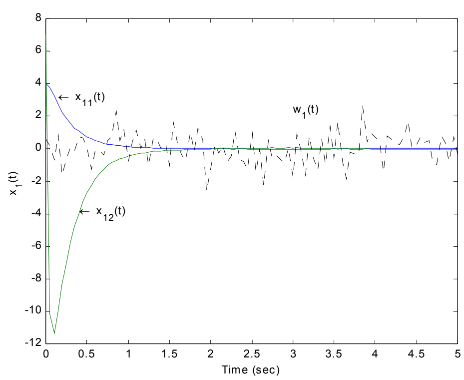

Fig. 3. The time response of the first subsystem in decoupled control design.

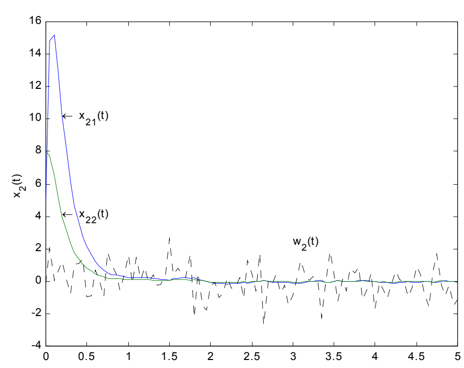

Fig. 4. The time response of the second subsystem in decoupled control design.

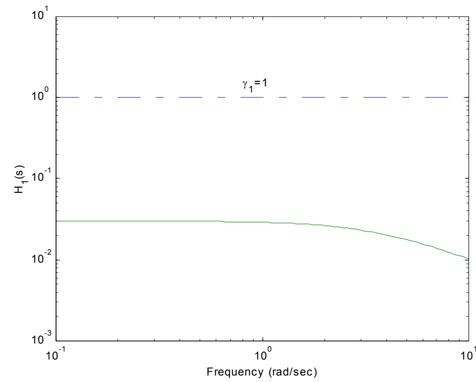

Fig. 5. The frequency response of the first subsystem in decentralized control design.

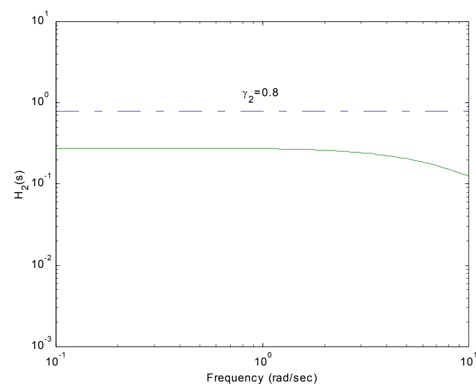

Fig. 6. The frequency response of the second subsystem in decentralized control design.

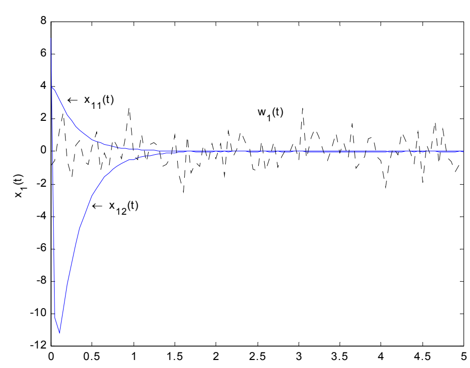

Fig. 7. The time response of the first subsystem in decentralized control design.

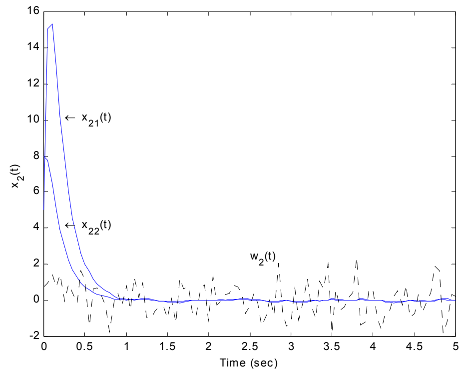

Fig. 8. The time response of the second subsystem in decentralized control design.

V. CONCLSION

In this paper, we have presented the multiobjective state-feedback control for stochastic large-scale systems via linear matrix inequality method. Three design approaches, the decoupled, centralized, and decentralized control, and associated multiobjective performance are investigated according to the structural characteristics of the considered stochastic large-scale systems.

In case the matching condition is satisfied, the overall system can be formulated and designed in terms of the individual subsystem via decoupled control approach. Though the individual performance specification can be derived, the state information from other subsystems may be needed for this representation. In the case that the matching condition is not satisfied, the centralized control approach considers the composed subsystems all together. Then, all state information of the whole system is assumed to be available for each individual subsystem; moreover, only the design specification in terms of overall system is derivable. The approach of decentralized control does not depend on the matching condition and uses only local state information to deliver performance for individual subsystem.

The numerical conditions based on LMI method are derived for the considered stochastic large-scale systems to achieve the desired multiobjective performance, the H_∞-norm disturbance attenuation level, state covariance upper bound, and disk regional pole placement. Finally, the proposed derivations are manifested by the numerical example.

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