Quality performance assessment system for Weibull lifetime product

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ABSTRACT

Process capability analysis has been developed for assessing quality performance. In practice, lifetime performance index C_L is a popular means to assess quality performance, where L is the lower specification limit. Most capability indices assume that the quality characteristic has a normal distribution. Nevertheless, the lifetime model of many products generally may possess a non-normal distribution which including exponential, Pareto or Weibull distribution and so forth. Moreover, censored samples may arise in life testing experiments. In this study, a new approach of analyzing non-normal quality data is proposed. Applying the power data transformation technology, this study constructs a maximum likelihood estimator (*MLE*) of C_L under the Weibull distribution with the right type II censored sample. The *MLE* of C_L is then utilized to develop the new hypothesis testing procedure in the condition of known L. The new hypothesis testing procedure is a quality performance assessment system. Moreover, the managers can then employ the new testing procedure to determine whether the quality performance of products adheres to the required level.

Keywords: Quality performance assessment system, Process capability analysis, lifetime performance index, Right type II censoring

韋伯壽命商品的品質績效評估系統

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摘　　要

製程能力分析被發展用以品質績效的評估。在實務上，壽命績效指標 C_L 是用以評估品質績效的受歡迎方法，其中 L 是規格下限。大多數製程能力指標都假設商品的品質特性是常態分配。然而很多商品的壽命模式是非常態分配，包括指數分配、柏拉圖分配、韋伯分配等等。並且在壽命測試實驗中，設限資料是經常發生的。爲了使用壽命績效指標 C_L 能更實際與準確地評估商品的壽命績效。本研究提出一個分析非常態品質資料的新方法。在商品的壽命模式是韋伯分配而且是右型 II 設限樣本的假設之下，利用冪次資料轉換技巧去建立一個 C_L 的最大概似估計式。然後此最大概似估計式用以發展一個新的假設檢定程序。而此新的假設檢定程序是一個品質績效評估系統，管理者利用這新的假設檢定程序去確定商品的品質績效是否達到規格要求，並且加強製程能力。

關鍵詞：品質績效評估系統、製程能力分析、壽命績效指標、右型 II 設限樣本

1. INTRODUCTION

Effectively managing and measuring the business operational process is widely seen as a means of ensuring business survival through reduced time to market, increased quality and reduced costs. Process capability analysis is utilized to assess whether product quality meets the required level. For instance, Montgomery (1985) (or Kane (1986)) proposed the process capability index C_L for evaluating the lifetime performance of electronic components, where L is the lower specification limit, since the lifetime of electronic components exhibits the larger-the-better quality characteristic of time orientation. Tong et al. (2002) constructed a uniformly minimum variance unbiased estimator (*UMVUE*) of C_L under an exponential distribution. Moreover, the *UMVUE* of C_L is then utilized to develop the hypothesis testing procedure. The managers can then employ the testing procedure to determine whether the lifetime of electronic components adheres to the required level. Hong et al. (2007) constructed a maximum likelihood estimator (*MLE*) of C_L under an pareto distribution. The *MLE* of C_L is then utilized to develop a new hypothesis testing procedure in the condition of known L. The new testing procedure can be employed by managers to assess whether the business performance adheres to the required level in the condition of known L.

All of the above process capability indices are derived under the assumption that the quality characteristic has a normal distribution, exponential distribution or pareto distribution. Nevertheless, these products should consist of manufactured goods such as electronic components, computers and clothing, as well as services such as the generation and distribution of electric energy, public transportation, banking, retailing and health care. Therefore, the lifetime model of many products generally may possess a non-normal distribution which including exponential, Pareto or Weibull distribution and so forth. To utilize the lifetime performance index C_L in assessing the lifetime performance of products more generally and accurately. In this paper, the Weibull distribution is to assess the lifetime performance of products. The Weibull distribution has a great variety of shapes and makes it extremely flexible in fitting data. Let the lifetime X of such a product has the Weibull distribution with the probability density function (p.d.f.) is

$$f_X(x) = \frac{\beta}{\alpha^{\beta}} x^{\beta-1} \exp[-(\frac{x}{\alpha})^{\beta}], x > 0, \alpha > 0, \beta > 0. \quad (1)$$

The parameter β is called the shape parameter, and the parameter α is called the scale parameter. For the special case β=1, the Weibull distribution is the simple exponential distribution. For the special case β=2, the Weibull distribution is the Rayleigh distribution. In addition, for 3≤β≤4, the shape of the Weibull distribution is close to that of the normal distribution (see Nelson (1982)). The shape parameters β determine the variety of shapes in the Weibull distribution and the failure rate function. But the effect of the scale parameter α is to only stretch out the graph. The β is more important than α in the Weibull distribution. We present a "shape-first" approach for the Weibull distribution that is to fit β before fitting α.

In life testing experiments, the experimenter may not always be in a position to observe the life times of all the products put on test. This may be because of time limitation and/or other restrictions (such as money, material resources, mechanical or experimental difficulties, etc.) on data collection. Therefore, censored samples may arise in practice. The issues of censored data are often ignored in knowledge discovery. In this paper, we consider the case of the type II right censoring. For the type II right censoring, only the first *r* lifetimes have been observed and the lifetimes for the remaining (*n-r*)components are unobserved or missing.

The main aim of this study will develop a quality performance assessment system for non-normal lifetime product. Apply the power data transformation technology to constructs a maximum likelihood estimator (*MLE*) of C_L under the Weibull distribution with the right type II censored sample. The *MLE* of C_L is then utilized to develop the hypothesis testing procedure in the condition of known L. The new hypothesis testing procedure is a quality performance assessment system for non-normal lifetime product. Moreover, the new testing procedure can be employed by managers to assess whether the lifetime of products adheres to the required level in the condition of known *L*.

The rest of this paper is organized as follows. Section 2 introduces some properties of the lifetime performance index for lifetime of products with the Weibull distribution based on the right type II censored sample, in addition, to discuss the relationship between the lifetime performance index and conforming rate. Section 3 then presents the maximum likelihood estimator of the lifetime performance index and its statistical properties. Section 4 develops a new hypothesis testing procedure for the lifetime performance index. Two numerical examples and concluding remarks are made in Section 5 and Section 6, respectively.

2. THT LIFETIME PERFORMANCE INDEX AND THE CONFORMING RATE

Suppose that the lifetime of products *X* may be modeled by a Weibull distribution with the p.d.f. as (1). Clearly, a longer lifetime implies a better product quality. Hence, the lifetime is a larger-the-better type quality characteristic. The lifetime is generally required to exceed *L* years to both be economically profitable and investors. Montgomery (1985) developed a capability index C_L for properly measuring the larger-the-better quality characteristic. C_L is defined as $C_L = \frac{\mu-L}{\sigma}$, where the process mean μ, the process standard deviation σ, and *L* is the lower specification limit.

To assess the lifetime performance of products, C_L can be defines as the lifetime performance index. Under the power data transformation $Y=X^{\beta}$, $\beta>0$(known), the distribution of is an exponential distribution and with p.d.f. is

$$f_Y(y,\theta) = \theta \exp(-\theta y), y > 0, \theta = \alpha^{-\beta} > 0 \qquad (2)$$

Moreover, there are several important properties, as follows:

- The lifetime performance index C_L can be rewritten as:

  $$C_L = \frac{\mu - L}{\sigma} = 1 - \theta L,\ C_L<1, \qquad (3)$$

  where the process mean μ=ey=$1/\theta$, the process standard deviation $\sigma = \sqrt{VARY}$ $=1/\theta$, and *L* is the lower specification limit.
- The cumulative distribution function of is given by:

$$F_Y(y) = 1 - e^{-\theta y}, \quad y > 0. \tag{4}$$

- The failure rate function is defined by:

$$r(y) = f_Y(y)/(1 - F_Y(y)) = \theta, \quad \theta > 0. \tag{5}$$

The important properties can be determined by using power data transformation $Y=X^{\beta}$, $X>0$, $\beta>0$ (known). And the data set of and transformed data set of have the same effect in assessing the lifetime performance of products. When the mean $1/\theta(>L)$, then $C_L>0$. From equations (3) and (5), we can see that the larger the mean $1/\theta$, the smaller the failure rate and the lager C_L. Therefore, the lifetime performance index C_L reasonably and accurately represents the lifetime performance of new product.

If the lifetime of a product X which $Y=X^{\beta}$, $\beta>0$ (known) exceeds the lower specification limit L, then the product is defined as a conforming product. The ratio of conforming products is known as the conforming rate, and can be defined as:

$$P_r = P(Y \geq L) = e^{C_L - 1}, \quad -\infty < C_L < 1. \tag{6}$$

Obviously, a strictly increasing and one to one relationship exists between conforming rate P_r and the lifetime performance index C_L. Therefore lifetime performance index C_L can be a flexible and effective tool, not only evaluating product performance, but also for estimating the conforming rate P_r.

3. MLE OF LIFETIME PERFORMANCE INDEX

In lifetime testing experiments of products, the experimenter may not always be in a position to observe the lifetimes of all the items on test due to time limitation and/or other restrictions (such as money, material resources, mechanical or experimental difficulties, etc) on data collection. Therefore, censored samples may arise in practice. In this paper, we consider the case of the right type II censoring.

With type II censoring, the value of r is chosen before the data are collected, and the data consist of the r smallest lifetimes in a random samples $X_1, X_2, \ldots, X_n$ from the Weibull distribution with p.d.f. (1), and $X_{(1)}<X_{(2)}<\ldots<X_{(r)}$, $r \leq n$ are the corresponding right type II censored sample. Since the joint p.d.f. of $X_{(1)}, X_{(2)}, \ldots, X_{(r)}$ is

$$\frac{n!\beta^r}{(n-r)!\alpha^{\beta r}}\left(\prod_{i=1}^{r} X_{(i)}^{\beta-1}\right)\exp\left[-\sum_{i=1}^{r}(X_{(i)}/\alpha)^{\beta} - (n-r)(X_{(r)}/\alpha)^{\beta}\right] \tag{7}$$

So, the log-likelihood function L is

$$\ln\frac{n!\beta^r}{(n-r)!\alpha^{\beta r}} + \sum_{i=1}^{r}\ln X_{(i)}^{\beta-1} - \sum_{i=1}^{r}\frac{X_{(i)}^{\beta}}{\alpha^{\beta}} - \frac{(n-r)X_{(r)}^{\beta}}{\alpha^{\beta}} \tag{8}$$

By $\frac{\partial \ln L}{\partial \alpha} = 0$ and $\theta = \alpha^{-\beta}$, hence, we show that the maximum likelihood estimator $\hat{\theta}$ of θ is given by

$$\hat{\theta} = \frac{r}{\sum_{i=1}^{r} X_{(i)}^{\beta} + (n-r)X_{(r)}^{\beta}}, \quad r \leq n. \tag{9}$$

By using the invariance of maximum likelihood estimator (see Zehna (1966)), the maximum likelihood estimator of C_L can be written as:

$$\begin{aligned}\hat{C}_L &= 1 - \hat{\theta}L \\ &= 1 - \frac{rL}{\sum_{i=1}^{r} X_{(i)}^{\beta} + (n-r)X_{(r)}^{\beta}}, \ r \leq n\end{aligned} \tag{10}$$

By the power data transformation $Y = X^{\beta}$, $\beta>0$, the maximum likelihood estimator of C_L can be rewritten as:

$$\hat{C}_L = 1 - \frac{rL}{\sum_{i=1}^{r} Y_{(i)} + (n-r)Y_{(r)}}, \ r \leq n. \tag{11}$$

We need the following Theorem 3.1. and Corollary 3.1. of Lawless (2003) to derive the properties of $\hat{C}_L$.

Theorem 3.1. Let $y_{(1)}<y_{(2)}<...<y_{(r)}$ be the first r ordered observations of a random sample of size n from exponential distribution and with p.d.f. as (2). Then $W_1,...W_r$ are independent and identically distributed, also with p.d.f. as (2), where , $W_i=(n-i+1)(y_{(i)}-y_{(i-1)})$, $i=1,2...,r$.

Corollary 3.1. Under the conditions of Theorem 3.1. $W=\sum_{i=1}^{r} y_{(i)}+(n-r)y_{(r)}$, has a distribution given by $2\theta W \sim \chi^2_{(2r)}$.

Let $W=\sum_{i=1}^{r} X_{(i)}^{\beta}+(n-r)X_{(r)}^{\beta}$, then by using Theorem 3.1. and Corollary 4.1.1. of Lawless (2003), we obtain that $2\theta W \sim \chi^2_{(2r)}$. Hence, the expectation of $\hat{C}_L$ can be derived as follows:

$$
\begin{aligned}
E\hat{C}_L &= E\left(1-\frac{rL}{\sum_{i=1}^{r} X_{(i)}^{\beta}+(n-r)X_{(r)}^{\beta}}\right) \\
&= E\left(1-\frac{rL}{W}\right) \qquad (12) \\
&= 1-2rL\theta E\left(\frac{1}{2\theta W}\right)=1-\frac{rL\theta}{r-1}.
\end{aligned}
$$

But $E\hat{C}_L \neq C_L$, where $C_L=1-\theta L$. Hence, the maximum likelihood estimator $\hat{C}_L$ is not an unbiased estimator of C_L. But when $r\rightarrow\infty$, $E\hat{C}_L \rightarrow C_L$ so the maximum likelihood estimator $\hat{C}_L$ is asymptotically unbiased estimator. Moreover, we also show that $\hat{C}_L$ is consistent.

4. TESTING PROCEDURE FOR THE LIFETIME PERFORMANCE IINDEX

Construct a statistical testing procedure to assess whether the lifetime performance index adheres to the required level. Assuming that the required index value of lifetime performance is larger than c, where denotes the target value, the null hypothesis H_0:$C_L \leq c$ and the alternative hypothesis H_1:$C_L > c$ are constructed. By using $\hat{C}_L$, the *MLE* of C_L as the test statistic, the rejection region can be expressed as $\{\hat{C}_L > C_0\}$. Given the specified significance level α^*, the critical value C_0 can be derived:

$$
C_0 = 1-\frac{2r(1-c)}{CHIINV(1-\alpha^*,2r)}, \qquad (13)
$$

where *c*, α^*, and *r*, *CHINV* $(1-\alpha^*, 2r)$ denote the target value, the specified significance level , the observed number and the lower 1-percentile of $\chi^2_{(2r)}$, respectively. Moreover, we also find that C_0 is independent of *n*.

Given the specified significance level α^*, the level $(1-\alpha^*)$ one-sided confidence interval for can be derived as follows:

With the pivotal quantity $2\theta W$, where $2\theta W \sim \chi^2_{(2r)}$ and *CHINV* $(1-\alpha^*, 2r)$ function which represents the lower $1-\alpha^*$ percentile of $\chi^2_{(2r)}$.

$$
\begin{aligned}
& P(2\theta W \leq CHIINV(1-\alpha^*,2r)) = 1-\alpha^*, \\
& \Rightarrow P\left(\frac{2r(1-C_L)}{1-\hat{C}_L} \leq CHIINV(1-\alpha^*,2r)\right) = 1-\alpha^*, \qquad (14) \\
& \Rightarrow P\left(C_L \geq 1-\frac{(1-\hat{C}_L)CHIINV(1-\alpha^*,2r)}{2r}\right) = 1-\alpha^*,
\end{aligned}
$$

where $C_L=1-\theta L$ and $\hat{C}_L = 1-\frac{rL}{W}$.

From Equation (14), then

$$
C_L \geq 1-\frac{(1-\hat{C}_L)CHIINV(1-\alpha,2r)}{2r} \qquad (15)
$$

is the level $(1-\alpha^*)$ one-sided confidence interval for C_L.

Thus, the level $(1-\alpha^*)$ lower confidence bound for C_L can be derived:

$$
\underline{LB} = 1-\frac{(1-\hat{C}_L)CHIINV(1-\alpha,2r)}{2r} \qquad (16)
$$

where $\hat{C}_L$, α^* and r denote the maximum likelihood estimator of C_L, the specified significance level and the observed number, respectively.

The managers can then employ the one-sided hypothesis testing to determine whether the lifetime performance index adheres to the required level. The proposed testing procedure about C_L can be organized as follows:

Step 1. By using the least squares method for various values of β and the "shape-first" approach to fit the optimal value β of and estimate α such that the Residual Sum of Squares (SSE) is minimized. Then, β is defined as known.

Step 2. The goodness of fit test based on the Gini statistic (see Gail and Gastwirth (1978)) for type II censored data.

Step 3. Let the power data transformation $Y_{(i)} = X_{(i)}^{\beta},\ \beta > 0, i = 1,2,\cdots,r$ for the type II right censored sample $X_{(1)}$, $X_{(2)}$, …,$X_{(r)}$.

Step 4. Determine the lower lifetime limit L for products and lifetime performance index value , then the testing null hypothesis H_0:$C_L \le c$ and the alternative hypothesis H_1:$C_L > c$ is constructed.

Step 5. Specify a significance level α.

Step 6. Calculate the value of test statistic by (11):

$$\hat{C}_L = 1 - \frac{rL}{\sum_{i=1}^{r} Y_{(i)} + (n-r)Y_{(r)}}, r \le n.$$

Step 7. Obtain the critical value C_0 by using (13), according to the target value c, the observed number r and the significance level α^*.

Step 8. The decision rule of statistical test is provided as follows:

"If $\hat{C}_L > C_0$, it is concluded that the lifetime performance index of product meets the required level."

Based on the proposed testing procedure, the lifetime performance of products is easy to assess. Moreover, the new hypothesis testing procedure is a quality performance assessment system for non-normal lifetime product. One numerical examples of the proposed testing procedure are given in the Section 6, and the numerical example illustrate the use of the testing procedure. In addition, the proposed testing procedure can be constructed with the one-sided confidence interval too. The decision rule of statistical test is "If performance index value $c \notin [\underline{LB}, \infty)$, it is concluded that the lifetime performance index of products meets the required level."

5. NUMERICAL EXAMPLES

In this section, we propose the new hypothesis testing procedure to two practical data sets. Example 1 considered is the failure data of n=12, r=10 electrical insulating fluids from Nelson (1982). Example 2 is the results of a life-test experiment in which n=24, r=17 electric cords for a small appliance are flexed by a test machine until failure, and it yields a reasonably straight plot on the normal paper (Nelson (1982)).The numerical examples illustrates the use of the proposed testing procedure. The proposed testing procedure not only can handle non-normal quality data, also can handle an approximate normal quality data.

Example 1. Nelson (1982, p. 124, Table 10. 3) presents the results of a life-test experiment in which specimens of a type of electrical insulating fluid were subject to a constant voltage stress. The length of time until each specimen failed (or brokedown) was observed at 30 Kv. The censored sample of the lifetime data of $n = 12$, $r = 10$ electric cords are 50, 134, 187, 882, 1448, 1468, 2290, 2932, 4138, 15750(seconds). Then, we also state the proposed testing procedure about C_L as following:

Step 1. Let X have the Weibull distribution with p.d.f. as (1). Then the cumulative distribution function of X is given as

$$F_X(x) = 1 - \exp[-(x/\alpha)^{\beta}],\ x > 0. \qquad (17)$$

The expectation of $F_X(x_{(i)})$ is

$$E[F_X(x_{(i)})] = i/(n+1)\ ,\ i = 1,2,\cdots,r,\ r \le n. \qquad (18)$$

y using the approximate equation is given as

$$\ln(1 - \frac{i}{n+1}) \approx -(\frac{x_{(i)}}{\alpha})^{\beta},\ i = 1,2,\cdots,r \qquad (19)$$

and the least squares method for various values of β and the "shape-first" approach to fit the optimal value of β and estimate such that the Residual Sum of Squares (SSE) is minimized.

The ranking data $\{x_{(i)},\ i = 1,2,\cdots,10\}$={3.912, 4.898, 5.231, 6.782, 7.279, 7.293, 7.736, 7.983, 8.338, 9.668}

The values of Residual Sum of Squares (SSE) and the estimation of α ($\hat{\alpha}$) for various values of β are shown in Figure 1. This display indicates that β=0.4 is very close to the optimum value 0.1437 and $\hat{\alpha}$=4930.68. Then, β is defined as known. In practice $0.3 \le \beta \le 0.5$, may be the candidates in SSE≤ 0.2362 criterion.

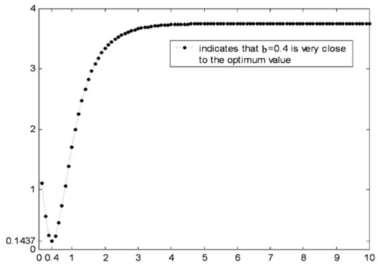

Figure 1: Plot of residual sum of squares SSE versus β

Step 2. The goodness of fit test based on the Gini statistic for type II right censored data. The operational lifetimes data of electrical insulating fluid $\{x_{(i)},\ i = 1,2,\cdots,10\}$ ($r = 10$, $n = 12$), and apply the Gini statistics (see Gail and Gastwirth (1978)) to the hypothesis that the data from the Weibull distribution with the p.d.f. is

$$f_X(x) = \frac{0.4}{4930.68^{0.4}} x^{-0.6} \exp[-(\frac{x}{4930.68})^{0.4}],\ x > 0. \qquad (20)$$

at level $\alpha^* = 0.05$.

The null hypothesis is H_0:X~ the Weibull distribution with the p.d.f. (20).

The Gini statistics

$$G_r = (\sum_{i=1}^{r-1} i W_{i+1}) / (r-1) \sum_{i=1}^{r} W_i \qquad (21)$$

where, $W_i = (n-i+1)(y_{(i)} - y_{(i-1)}),\ i = 1,2,...,r,\ y_0 = 0)$, and the power data transformation $y_{(i)} = x_{(i)}^{\beta}$, β>0.

For r=3,...,20, the rejection region $\{G_r > \xi_{1-\alpha^*/2}$ or $G_r < \xi_{\alpha^*/2}\}$, where the critical value $\xi_{\alpha^*/2}$ is the $\alpha^*/2$ percentage points of the G_r statistic. For β=0.4, the Gini statistics

$$G_{10} = (\sum_{i=1}^{9} i W_{i+1}) / (10-1) \sum_{i=1}^{10} W_i = 0.46123.$$

$\therefore \xi_{0.025} = 0.31232 < G_{10} = 0.46123 < \xi_{0.975} = 0.68768$, so we can not reject H_0 at the 0.05 level of significance. We can conclude the operational lifetimes data of electrical insulating fluid from the Weibull distribution with the p.d.f. (20) at level $\alpha^* = 0.05$.

Step 3. The operational lifetimes data of electrical insulating fluid $\{x_{(i)},\ i = 1,2,\cdots,10\}$ and we take the power transformation of $y(i) = x_{(i)}^{0.4}$, i =1,2,...,10.

Step 4. The lower lifetime limit L is assumed to be 3.0, i.e. if the lifetime of an electrical insulating fluid exceeds 15.6 seconds, then the electrical insulating fluid is defined as a conforming product. To deal with the product managers' concerns regarding operational performance, the conforming rate P_r of operational performances is required to exceed 80 percent. By using (6), the C_L value operational performances is required to exceed 0.80. Thus, the performance index value is set at c = 0.80. The testing hypothesis $H_0 : C_L \leq$ 0.80 v.s. $H_1 : C_L \leq 0.80$ is constructed.

Step 5. Specify a significance level α^*= 0.05.

Step 6. Calculate the value of test statistic

$$\hat{C}_L = 1 - \frac{10\times 3}{194.072 + (12-10)\times 47.74} = 0.896.$$

Step 7. Obtain the critical value C_0= 0.873 by using (13), according to c= 0.80, r=10 and the significance level α^*= 0.05.

Step 8. Because of $\hat{C}_L$ = 0.896 > C_0= 0.873, so we do reject to the null hypothesis H_0: $C_L \leq 0.80$. Thus, we can conclude that the lifetime performance index of electrical insulating fluid operation does meet the required level.

Example 2. Nelson (1982, p. 121, Table 10.1) presents the results of a life-test experiment in which electric cords for a small appliance are flexed by a test machine until failure. The test simulates actual use, but highly accelerated. Each week, 12 cords go on the machine and run a week. After a week, the un-failed cords come off test make room for a new sample of cords. The censored sample of the lifetime data of n = 24, r =17 electric cords is given as following, and it yields a reasonably straight plot on the normal paper: $\{X_{(i)}, i = 1,2,\cdots, 17\}$={57.5, 77.8, 88.0, 96.9, 98.4, 100.3, 100.8, 102.1, 103.3, 103.4, 105.3, 105.4, 122.6, 139.3, 143.9, 148.0, 151.3(hours)}.

Then, we also state the proposed testing procedure about C_L as following:

Step 1. Let have the Weibull distribution with p.d.f. as (1). Then the cumulative distribution function of is given as (17). The expectation of β is given as (18). By using the approximate equation is given as (19) and the least squares method for various values of β and the "shape-first" approach to fit the optimal value of β and estimate α such that the Residual Sum of Squares (*SSE*) is minimized.

The ranking data $\{x_{(i)}, i=1,2,\cdots, 17\}$= {57.5, 77.8, 88.0, 96.9, 98.4, 100.3, 100.8, 102.1, 103.3, 103.4, 105.3, 105.4, 122.6, 139.3, 143.9, 148.0, 151.3(hours)}.

The values of Residual Sum of Squares (*SSE*) and the estimation of $\alpha(\hat{\alpha})$ for various values of β are shown in Figure 2. This display indicates that β= 2.7 is very close to the optimum value 0.19656 and $\hat{\alpha}$=145.62. Then, β is defined as known. In practice, $2.4 \leq \beta \leq 3.1$ may be the candidates in $SSE \leq 0.22$ criterion.

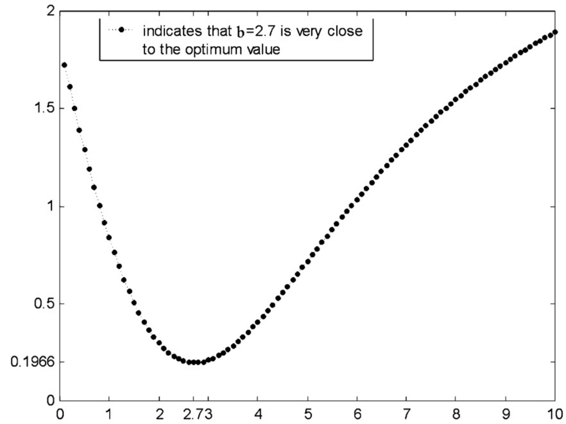

Figure 2: Plot of residual sum of squares SSE versus β

Step 2. The goodness of fit test based on the Gini statistic for type II right censored data. The operational lifetimes data of electrical insulating fluid $\{X_{(i)}, i =1,2,\cdots, 17\}$ (r = 17, n =24), and apply the Gini statistics (see Gail and Gastwirth (1978)) to the hypothesis that the data from the Weibull distribution with the p.d.f. is

$$f_X(x) = \frac{2.7}{145.62^{2.7}} x^{1.7} \exp[-(\frac{x}{145.62})^{2.7}], x > 0. \quad (22)$$

at level α^*=0.05.

The null hypothesis is H_0 : X~ the Weibull distribution with the p.d.f. (22).

The Gini statistics

$$G_r = (\sum_{i=1}^{r-1} iW_{i+1}) / (r-1) \sum_{i=1}^{r} W_i \quad (23)$$

where $Wi=(n-i+1)(y_{(i)}-y_{(i-1)}$, i=1,2,,..,r, $y_{(0)}$=0, and the power data transformation $y_{(i)}=x_{(i)}^{\beta}$, β>0.

For r =3,...,20, the rejection region $\{G_r > \xi_{1-\alpha^*/2}$ or $G_r < \xi_{\alpha^*/2}\}$, where the critical value $\xi_{\alpha^*/2}$ is the $\alpha^*/2$ percentage points of the G_r statistic. For β=27, the Gini statistics

$$G_{17} = (\sum_{i=1}^{16} iW_{i+1}) / (17-1) \sum_{i=1}^{17} W_i = 0.44807.$$

$\therefore\ \xi_{0.025} = 0.35893 < G_{17} = 0.44807 < \xi_{0.975} = 0.64107$, so we can not reject H_0 at the 0.05 level of significance. We can conclude the operational lifetimes data of electric cords from the Weibull distribution with the p.d.f. (22) at level α=0.05.

Step 3. The operational lifetimes data of electric cords $\{x_{(i)}, i = 1,2,\cdots,17\}$ and we take the power transformation of $y_{(i)} = x_{(i)}^{2.7}$, i =1, 2,⋯,17.

Step 4. The lower lifetime limit L is assumed to be 69294.18, i.e. if the lifetime of an electric cord exceeds 62.1 hours, then the electric cord is defined as a conforming product. To deal with the product managers' concerns regarding operational performance, the conforming rate P_r of operational performances is required to exceed 80 percent. By using (6), the C_L value operational performances is required to exceed 0.80. Thus, the performance index value is set at c = 0.80. The testing hypothesis $H_0: C_L \leq 0.80$ v.s. $H_1: C_L > 0.80$ is constructed.

Step 5. Specify a significance level α^*= 0.05.

Step 6. Calculate the value of test statistic

$$\hat{C}_L = 1 - \frac{17 \times 69294.18}{1.37751 \times 10^{59} + (24-17) \times 7.17218 \times 10^{58}}$$
$$= 0.999.$$

Step 7. Obtain the critical value C_0 = 0.860 by using (13), according to c = 0.80, r = 17 and the significance level α^* = 0.05.

Step 8. Because of $\hat{C}_L$ = 0.999 > C_0 = 0.860, so we do reject to the null hypothesis H_0 : $C_L \leq 0.80$. Thus, we can conclude that the lifetime performance index of electric cord operation does meet the required level.

6. CONCLUSIONS

Process capability indices are widely employed by manufacturers to assess the performance and potential of their processes. Most process capability indices have been developed or investigated to assess whether product quality meets the required level under a normal lifetime model. Nevertheless, the lifetime model of many products generally may possess a non-normal distribution. Moreover, in life testing experiments, the experimenter may not always be in a position to observe the life times of all the businesses (or items) put on test. This may be because of time limitation and/or other restrictions (such as money, material resources, mechanical or experimental difficulties, etc.) on data collection. Therefore, censored samples may arise in practice. So, we consider the case of the right type II censoring, mator (MLE) of C_L under the Weibull distribution with the right type II censored sample. The *MLE* of C_L is then utilized to develop the hypothesis testing procedure in the condition of known *L*. Moreover, the new hypothesis testing procedure is a quality performance assessment system for non-normal lifetime product. The proposed testing procedure is easily applied and can effectively evaluate whether the lifetime of products meets requirements. The managers can also utilize this procedure to enhance process capability.

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