Vibration Control for a Suspension System with Magnetorheological Damper

運用磁流變液阻尼於懸吊系統之減震控制

Kun-Yung Chen[1,*] Yi-Wei Chen[1]
陳崑永[1,] 陳一葳[1]
[1]Department of Mechanical Engineering, Air Force Institute of Technology
[1] 空軍航空技術學院機械工程科

Abstract

The new dynamic modeling and vibration control for the suspension system with magnetorheological (MR) damper is formulated and studied in this paper. The LuGre model is implemented to fit for the hysteresis phenomenon for the MR damper. The new dynamic model of the suspension system is successfully formulated with the LuGre model and electric current controlled by the input voltage. For the full available outputs of the suspension system, the vibration control strategies are proposed by using sliding mode control (SMC). According to the energy conservation of the suspension system, an adjustable desired current state is proposed to absorb the external input energy to isolate the vibration. Finally, there are three kinds of tire deflection are verified to test the vibration control performance for the SMC. From the simulation results, the proposed SMC can practically apply to the full outputs to restrain the acceleration of the sprung mass of the suspension system.

Keywords: Hysteresis; LuGre model; Magnetorheological (MR) damper; Suspension system.

摘要

本篇論文主要探討安裝磁流變液阻尼器於懸吊系統之動力系統數學建模與震動控制之研究。磁流變液本身具有非線性的遲滯現象，本論文運用 LuGre 模式描述該非線性的遲滯現象，另使用力學平衡建立懸吊系統的機械方程式，電路學平衡建立磁流變液阻尼器電學方程式。由上述機械方程式與電學方程式，懸吊系統之動力系統數學模式即可以狀態空間矩陣方程式描述。系統輸出(位移、速度、電流)均可完整量測獲得情況下，本研究運用順滑模態控制於懸吊系統的震動控制，以可調適參考電流為控制目標吸收震動能量之方法執行減震控制。為測試本研究提出之震動控制方法，在理論模擬中，提出三種不同形式外在震動干擾，本研究所提方法均可快速將懸吊系統震動幅度與加速度降低。

關鍵字：遲滯現象、 磁流變液阻尼器、懸吊系統

1 Introduction

The suspension system is generally applied in the dynamic system to isolate and absorb the external vibration. It is widely installed in the motorcycle, vehicle, civil engineering, landing gear or other mechanisms to isolate the external vibration. The suspension system [1-4] has three kinds of the type including the passive, semi-active and active systems. The semi-active suspension system equipped with a magnetorheological (MR) damper is widely applied to perform the vibration control. The damping force of MR damper is adjusted by using input current. In the practical application for the MR damper, the nonlinear hysteresis phenomenon is the main drawback. There are four nonlinear modeling perspectives are discussed by Rebecca *et al.* [5] including nonlinear Bingham plastic model, viscous model, nonlinear hysteretic viscous model, and viscoelastic plastic model. The performances of the model are evaluated by errors of calculating equivalent viscous damping and force-time history between the model and experimental data. The theoretical and experimental studies are performed by Sassi *et al.* [6] for the design, development, and testing of a completely new MR fluid damper model that is employed for the semi-active control of automotive suspensions. The experimental and numerical study for the MR damper in the studies [7-9] by using the finite element method. The finite element model is formulated to examine and investigate the 2- D axis-symmetric MR damper. The studies [10-11] presented the results of the experimental and numerical analysis developed to study the nonlinear hysteretic responses of MR damper. Besides, Zhu *et al.* [11] proposed a review on structure design and analysis for the MR damper. Then, a rotary MR damper [12] with a specified damping torque capacity is designed and validated. An unsaturated magnetic flux density and high magnetic field intensity are discussed for the vehicle suspension system. Some mathematical models are proposed including the Squeeze Mode [13], Bingham plastic model and Herschel Bulkley model [14], Bouc-Wen model and modified Bouc-Wen model [15-16] to characterize the behavior of the MR damper.

A suspension system equipped with MR damper is a general semi-active suspension system. It can closed-loop adjust the viscous coefficient of MR damper to isolate the external vibration. The suspension system equipped with MR damper had been widely applied in motorcycle [17], vehicle [18], and railway vehicles [19]. It can offer the well riding comfortably for the passenger in the vehicle by controlling the properties of MR damper. The vibration control strategies [20-22] for the suspension system equipped with MR damper had been studied previously. A Bouc-Wen model and modified Bouc-Wen model is adopted to characterize the performance of the MR damper. However, the evolutionary variable for the Bouc-Wen model is an internal state that is unmeasurable, the previous studies seem not to discuss and deal with it.

In this paper, the LuGre model is implemented to characterize the hysteresis behavior of MR damper and an electric equation is applied to combine in the LuGre model to adjust the damping force with a voltage input. Then, a new dynamic model with the state-space form for the suspension system is successfully formulated. For the full available outputs of the suspension system, a sliding mode control (SMC) is proposed to perform the vibration control. From the simulation results, the proposed SMC perform well vibration control characteristic to restrain the acceleration of sprung mass of the suspension system.

2 Dynamic Modelling of the Suspension System

2.1 Modelling of the MR damper

The LuGre mathematical model had been implemented to fit for the hysteresis behavior of the MR damper, which is written as

$$\dot{z} = v - \alpha'|v|z, \tag{1}$$

$$f_{mr} = (-p_1 i^2 + p_2 i + p_3')v + \delta x + \varepsilon \dot{z} + (q_1 i + q_2)z, \tag{2}$$

where z is the internal state, v is the external input excitation velocity, f_{mr} is the exerted damping force, i is the input current and $i \geq 0$, α', δ, ε, p_{1-3} and $q_{1\text{-}2}$ are the parameters of MR damper.

Equations (1)-(2) are the mathematical model with LuGre model for the MR damper. By inserting Eq. (1) into Eq. (2), one can obtain the f_{mr}

$$f_{mr} = (-p_1 i^2 + p_2 i + p_3)v + (q_1 i + q_2 - \alpha|v|)z, \tag{3}$$

where $p_3 = p_3' + \varepsilon$ and $\alpha = \varepsilon\alpha'$.

Observing Eq. (3), it can be found that the damping force f_{mr} is affected by the input current, velocity, and internal state. Therefore, a completed electrical equation includes input voltage, resistance, inductor and electromotive force (EMF) which can be written as

$$u = iR + L\frac{di}{dt} + \lambda_d v, \tag{4}$$

where u is the input voltage, R is the resistance and L is the inductor, and λ_d is the EMF constant.

2.2 Modelling of quarter car suspension system

The quarter car model of suspension system with a MR damper is shown in Fig. 1. For the dynamic balancing equation of the suspension system, there are two subsystems, one is the suspension subsystem, and the other is the tire subsystem. Thus, the force balance equation for the suspension subsystem can be found as

$$f_s + f_{mr} = (m_B + \Delta m_B)\ddot{x}_B, \tag{5a}$$

where the spring force f_s is $f_s = k_s x_2$ and the damping force f_{mr} is

$$f_{mr} = (-p_1 i^2 + p_2 i + p_3)\dot{x}_2 + (q_1 i + q_2 - \alpha|\dot{x}_2|)z$$

The dynamic balancing equation for the tire subsystem can be found as

$$f_t - f_s - f_{mr} = m_W \ddot{x}_W, \tag{5b}$$

where tire force $f_t = k_t x_1$.

The details of Eqs. 5 (a-b) is written as

$$\ddot{x}_B = \frac{k_s}{m_B}x_2 + \frac{1}{m_B}(-p_1 i^2 + p_2 i + p_3)\dot{x}_2 + \frac{1}{m_B}(q_2 - \alpha|\dot{x}_2|)z + \frac{1}{m_B}q_1 iz - \frac{1}{m_B}\Delta m_B \ddot{x}_B, \tag{6a}$$

$$\ddot{x}_W = \frac{k_t}{m_W}x_1 - \frac{k_s}{m_W}x_2 - \frac{1}{m_W}(-p_1 i^2 + p_2 i + p_3)\dot{x}_2 - \frac{1}{m_W}(q_2 - \alpha|\dot{x}_2|)z - \frac{1}{m_W}q_1 iz \tag{6b}$$

Subtracting Eq. (6a) from Eq. (6b), one can obtain

$$\dot{v}_2 = \ddot{x}_W - \ddot{x}_B = -\rho k_s x_2 - \rho(-p_1 i^2 + p_2 i + p_3)v_2 - \rho(q_2 - \alpha|v_2|)z - \rho q_1 iz + \frac{1}{m_B}\Delta m_B \ddot{x}_B + \frac{k_t}{m_W}x_1, \tag{7}$$

where $v_2 = \dot{x}_W - \dot{x}_B$ and $\rho = (m_W + m_B)/(m_W m_B)$. Observing Eq. (7), one can find that x_2, v_2, z and i are the states of the suspension system equipped with MR damper, the tire deflection x_1 is the unknown external disturbance for the suspension system. Therefore, a state-space matrix form is formulated for the suspension system by using Eqs. (1), (4) and (7)

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}u + \mathbf{D}, \tag{8}$$

where $\mathbf{X} = [x_2 \quad v_2 \quad z \quad i]^T$,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\rho k_s & -\rho(-p_1 i^2 + p_2 i + p_3) & -\rho(q_2 - \alpha|v_2|) & -\rho q_1 z \\ 0 & 1 & -\alpha'|v_2| & 0 \\ 0 & -\lambda/L & 0 & -R/L \end{bmatrix},$$

$\mathbf{B} = [0 \quad 0 \quad 0 \quad 1/\mathrm{L}]^T$,

$\mathbf{D} = [0 \quad (\Delta m_B / m_B)\ddot{x}_B + (k_t/m_W)x_1 \quad 0 \quad 0]^T$, and u is input voltage. Observing the state of $x_2 = x_W - x_B$ in Eq. (8), one can find that the suspension deflection is compressed for $x_2 > 0$, and suspension deflection is stretched for $x_2 < 0$. When $x_2 = 0$, the suspension system is in the balance position. In additional, the x_1 and Δm_B are the unknown external disturbance for the suspension system. Therefore, the vibration control objects are to keep the states $x_2 = 0$ and $v_2 = 0$ to reduce vibration by using MR damper to reject the road displacement.

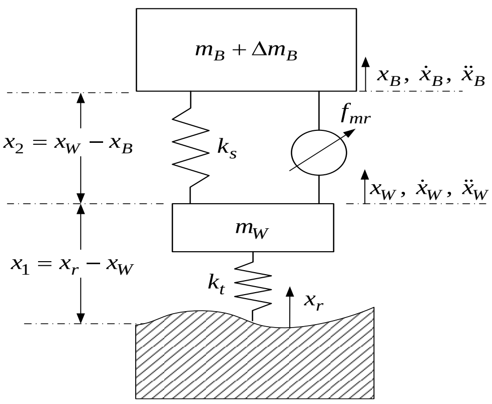

Figure 1 Quarter car suspension system

3 Design of a Controller

The dynamic model of the suspension system with MR damper is successfully formulated in Section 2. It is found that the states are the suspension displacement x_2, velocity v_2, internal state z, and current i. For the ideal control condition, it is assumed that the states of suspension system are full available and the SMC is proposed to perform the vibration control.

3.1 Sliding mode control (SMC)

In an ideal condition for the control application, the states of the suspension system are all available. The states of x_2, v_2, z and i are available which can be used in the SMC. Furthermore, the external and uncertainty variables $\mathbf{D}$ are unknown and bound.

3.1.1 Design of the SMC

Let a nominal model for the suspension system model Eq. (8) with an equivalent control input u_{eq} be

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}u_{eq}. \tag{9}$$

Let the sliding surface function s be

$$s = \boldsymbol{\sigma}(\mathbf{X}^* - \mathbf{X}), \tag{10a}$$

where $\boldsymbol{\sigma} \in \mathbf{R}^{1\times 4}$ is a known constant vector and $\mathbf{X}^* = [x_2^* \quad v_2^* \quad z^* \quad i^*]^T \in \mathbf{R}^{4\times 1}$ is the desired vector, x_2^*, v_2^*, z^* and i^* are the desired trajectories for x_2, v_2, z and i, respectively.

The equivalent control input u_{eq} can be obtained with the dynamics in the sliding mode $\dot{s} = 0$. Therefore, the differential sliding surface function s with respective to time is

$$\dot{s} = \boldsymbol{\sigma}(\dot{\mathbf{X}}^* - \dot{\mathbf{X}}) = \boldsymbol{\sigma}(\dot{\mathbf{X}}^* - \mathbf{A}\mathbf{X} - \mathbf{B}u_{eq}) = 0. \tag{10b}$$

Then, the equivalent control input u_{eq} can be found

$$u_{eq} = (\boldsymbol{\sigma}\mathbf{B})^{-1}\left[\boldsymbol{\sigma}(\dot{\mathbf{X}}^* - \mathbf{A}\mathbf{X})\right]. \tag{11}$$

To deal with the external and uncertainty variables $\mathbf{D}$ for the suspension system, a nonlinear vibration control input u_n is proposed to add with the equivalent control input u_{eq} in the control law. Then, the SMC control law u is proposed as

$$u = u_{eq} + u_n. \tag{12}$$

Let a Lyapunov function candidate with the sliding surface function s be

$$V = \frac{1}{2}s^2. \tag{13a}$$

Differentiating the Lyapunov function candidate with respective time, one can find that

$$\dot{V} = s\dot{s} = s\boldsymbol{\sigma}(\dot{\mathbf{X}}^* - \dot{\mathbf{X}}) = s\boldsymbol{\sigma}\left[\dot{\mathbf{X}}^* - \mathbf{A}\mathbf{X} - \mathbf{B}(u_{eq} + u_n) - \mathbf{D}\right]. \tag{13b}$$

By inserting u_{eq} in Eq. (11) into Eq. (13b), one can obtain

$$\dot{V} = -s\left[\boldsymbol{\sigma}\mathbf{B}u_n + \boldsymbol{\sigma}\mathbf{D}\right]. \tag{13c}$$

Let u_n be

$$u_n = (\boldsymbol{\sigma}\mathbf{B})^{-1}k_1 \cdot \mathrm{sgn}(s), \tag{14}$$

The $\boldsymbol{\sigma}\mathbf{D}$ is unknown but bound, and $0 \le |\boldsymbol{\sigma}\mathbf{D}| \le f$, f is known. The $\dot{V}$ in Eq. (13c) can be rewritten as

$$\dot{V}=-s\left[\boldsymbol{\sigma}\mathbf{B}u_n+\boldsymbol{\sigma}\mathbf{D}\right]=-s\left[k_1\,\mathrm{sgn}(s)+\boldsymbol{\sigma}\mathbf{D}\right]\le -k_1|s|-sf.$$

If k_1 is given as $k_1 \ge f$, one can find that

$$\dot{V}\le -k|s|-sf\le -k|s|\le 0. \tag{15b}$$

Therefore, the differential Lyapunov function candidate $\dot{V}\le 0$, the sliding surface function s is asymptotically stable and the system output $\mathbf{X}$ can converge to the desired trajectory $\mathbf{X}^*$. To reduce the chattering phenomena in the SMC, a saturation function is implemented to replace the sign function $\mathrm{sgn(s)}$ as

$$\mathrm{sgn}(s)=sat(\mathrm{s}/\varepsilon), \tag{16}$$

where $\mathrm{sat(s/\varepsilon)}=\begin{cases}1 & ,\ \mathrm{s}>\varepsilon,\ \varepsilon>0\\ \mathrm{s}/\varepsilon & ,-\varepsilon\le \mathrm{s}\le\varepsilon,\ \varepsilon>0.\\ -1 & ,\ \mathrm{s}<-\varepsilon,\ \varepsilon>0\end{cases}$

Finally, the proposed SMC for the suspension system with MR damper is found as

$$u=u_{eq}+u_n=\left(\boldsymbol{\sigma}\mathbf{B}\right)^{-1}\left[\boldsymbol{\sigma}(\dot{\mathbf{X}}^*-\mathbf{AX})+k_1\cdot \mathrm{sat}(s/\varepsilon)\right], \tag{17}$$

where $u=\begin{cases}u_{\max} & ,\ \text{if } u\ge u_{\max}\\ 0 & ,\ \text{if } u\le 0\end{cases}$, $u_{\max}$ is the maximum input voltage.

3.1.2 Design of the desired trajectories

In the sliding surface function, $\mathbf{X}^*$ is the desired vector x_2^*, v_2^*, z^* and i^* are the desired trajectories. In the control theory, the desired trajectory is the main control object for a dynamic system. In the suspension system, the control objects are to keep the sprung mass in a balance position ($x_2=0$ and $v_2=0$, and $z=0$). Therefore, the desired trajectories of x_2^*, v_2^*, z^* are assigned as $x_2^*=0$, $v_2^*=0$ and $z^*=0$, respectively. For the current state i, if the desired trajectory is set as $i^*=0$, the general desired vector of SMC for the suspension system is

$$\mathbf{X}^*=\mathbf{X}_0^*=\begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}^T\in\mathbf{R}^{4\times 1}. \tag{18}$$

From the above desired vector, it means that the damping force of MR damper is always kept in the minimum value. The damping force could not eliminate the external disturbance from the road displacement. To reduce the oscillation of the suspension system, the desired current $i^*=0$ is not a suitable value for the vibration control. The desired current i^* should be designed as an adjustable variable according to the system states to produce a adjustable damping force by the MR damper.

From the dynamic balancing equation of Eq. (7), it can be re-written as

$$\rho' k_t x_1 = pv_2 + qz + k_s x_2 + m_W \rho' \dot{v}_2, \tag{19}$$

It can be found that $\rho' k_t x_1$ is the external input force of the tire, pv_2 is the damping force of MR damper, qz is the internal force of MR damper, $k_s x_2$ is the spring force and $m_W\rho'\dot{v}_2$ is the output force. If the suspension system has a small deflection Δx_2 driven by the external tire force. The energy balance equation can be found as

$$\rho' k_t x_1\Delta x_2 = pv_2\Delta x_2 + qz\Delta x_2 + k_s x_2\Delta x_2 + m_W\rho'\dot{v}_2\Delta x_2 \quad (15a)$$

(20a)

The work done by external tire force is obtained by integrating Eq. (20a) as

$$E_i = E_d + E_p + E_k, \tag{20b}$$

where E_i is the input energy, E_d is the dissipation energy, E_p is the potential energy, E_k is the kinematic energy. The details of the above energy equation can be obtained as

$$E_i=\rho' k_t x_1\Delta x_2=\rho' k_t x_1\int dx_2=\rho' k_t x_1 x_2, \tag{21a}$$

$$\mathrm{E}_d = p\int v_2 dx_2 + qz\int dx_2 = pv_2^2 t + qzx_2, \tag{21b}$$

$$E_p = k_s x_2\Delta x_2 = k_s\int x_2 dx_2 = \frac{1}{2}k_s x_2^2, \tag{21c}$$

$$E_k = m_W\rho'\dot{v}_2\Delta x_2 = m_W\rho'\int\frac{dv_2}{dt}dx_2=\frac{1}{2}m_W\rho' v_2^2. \tag{21d}$$

When the external input energy E_i apply to the suspension system, only the E_d absorb the external input energy by changing p and q function. For an ideal condition of energy consumption, one can obtain a relationship for the energy balance equation as

$$E_i = E_d \tag{22}$$

were $E_p=0$, $E_k=0$, and E_d can completely absorb the external input energy E_i .

When the tire force $\rho' k_t x_1$ apply on the wheel, the relationship between the input energy and suspension deflection x_2 can be found as

$$E_i=\rho' k_t x_1 x_2\propto x_2, \tag{23a}$$

For the dissipation energy E_d with $z\approx 0$, a relationship can be fund as

$$E_d = pv_2^2 t + qzx_2 \propto pv_2^2 t\propto p\propto i\ge 0. \tag{23b}$$

When the external input energy E_i apply to the suspension system, the E_d should completely absorb the external input energy. From the relationship of Eqs. 23 (a) and (b), one can propose the desired current i^* as

$$i^*=\lambda|x_2|, \tag{24a}$$

where λ is a given constant.

According to the energy conservation law, the proposed desired vector of SMC for the suspension system is

$$\mathbf{X}^*=\mathbf{X}_1^*=\begin{bmatrix}0 & 0 & 0 & \lambda|x_2|\end{bmatrix}^T\in\mathbf{R}^{4\times 1}. \tag{25}$$

Finally, for the passenger's comfort, the acceleration $\ddot{x}_B$ of the vehicle is an obvious quantity for the motion and vibration of the car body. Thus, the criterion function of the normalized body acceleration for the quarter-car model is chosen as

$$J=\sqrt{\frac{1}{T}\int_{t=0}^{T}\left(\frac{\ddot{x}_B}{g}\right)^2 dt}, \tag{26}$$

where T is the total period of time and $g=9.8\ \mathrm{m/s^2}$ is the gravitational acceleration.

4 Numerical Simulations

The vibration control by using SMC for the suspension system with MR damper is proposed and performed numerically. There are two conditions including the full and some parts of available outputs. For the full available output, the SMC is proposed to reduce the acceleration of the car. In the numerical experiments, the suspension system with MR damper by using SMC is performed and verified. The numerical parameters are given in Table 1 to implement in MTALAB program. The displacement is the unknown external disturbance, and there are three Cases for are implemented to verify the vibration control. The three cases are shown as follows

Case 1: $x_1 = 0$ for $0 \le t < 1$, $x_1 = 0.2\sin 4\pi t$ m for $1 \le t \le 3$, and $x_1 = 0$ for $3 < t \le 10$.

Case 2: $x_1 = 0.1\sin 3\pi t + 0.05\cos \pi t$ m for $0 \le t \le 10$,

Case 3: $x_1 = a$ square signal for $0 \le t \le 10$.

4.1 MR damper and SMC

The numerical responses of the MR damper can be produced with the parameters (p_1, p_2, p_3', q_1, q_2, α', δ and ε) in Table 1 by using the different input velocities and current.

Table 1 Numerical parameters

$p_1 = 4$ N-s/mm$\cdot$A^2	$\alpha' = 0.7$ mm^{-1}	$\lambda = 5\times10^{-5}$ Vs/mm
$p_2 = 50$ N-s/mmA	$\delta = 3$ N/mm	$m_B = 40$ kg
$p_3' = 10$ N-s/mm	$\varepsilon = 0.5$ N-s/mm	$m_W = 5$ kg
$q_1 = 80$ N/mm$\cdot$A	$R = 4\ \Omega$	$k_s = 250$ N/mm
$q_2 = 10$ N/mm	$L = 0.1$ H	$k_t = 350$ N/mm

4.1.1 Numerical results

From Eqs. (1)-(2), one can find the responses of f_{mr} and z shown in Fig. 2. In the relationships between damping force f_{mr} and input velocity v, the more input current is supplied, the more damping force is obtained shown in Fig. 2 (a). In Fig. 2 (b), the responses of the internal state z is between in the small range. From the responses of Fig. 2, the LuGre model can properly describe the hysteresis phenomenon of the MR damper. Secondly, the proposed SMC with full available output is performed for the suspension system with MR damper to restrain the vibration and acceleration of vehicle. The parameters of the suspension system are given in Table 1, and the maximum input voltage is $u_{\max} = 30$ V. The displacement x_r of road applied into the tire is unknown and stochastic, the disturbance vector in Eq. (8) is also uncertainty. There are three Cases with x_1 are implemented to verify the vibration control with the proposed SMC. Additionally, the different desired trajectories ($\mathbf{X}_0^*$ and $\mathbf{X}_1^*$) are implemented in the proposed SMC to be compared. Figure 4 shows the responses of the proposed SMC with the desired trajectories $\mathbf{X}_0^*$ and $\mathbf{X}_1^*$ for Case 1. The x_1 is given as $x_1 = 0.2\sin 4\pi t$ m for $1 \le t \le 3$ shown in Fig. 3 (a). The responses of state of the suspension system by the

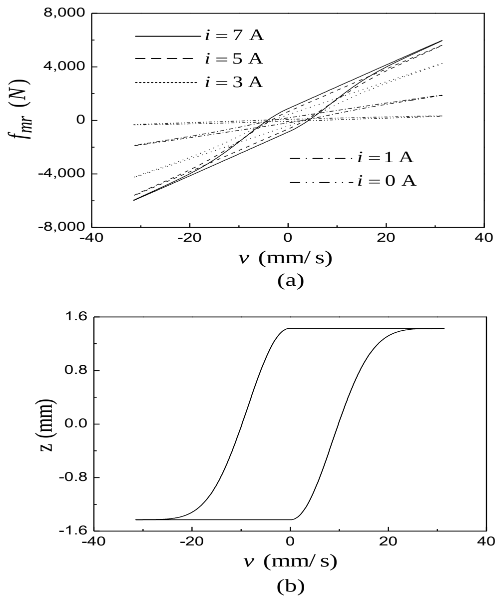

Figure 2 Responses of the MR damper with the input velocity

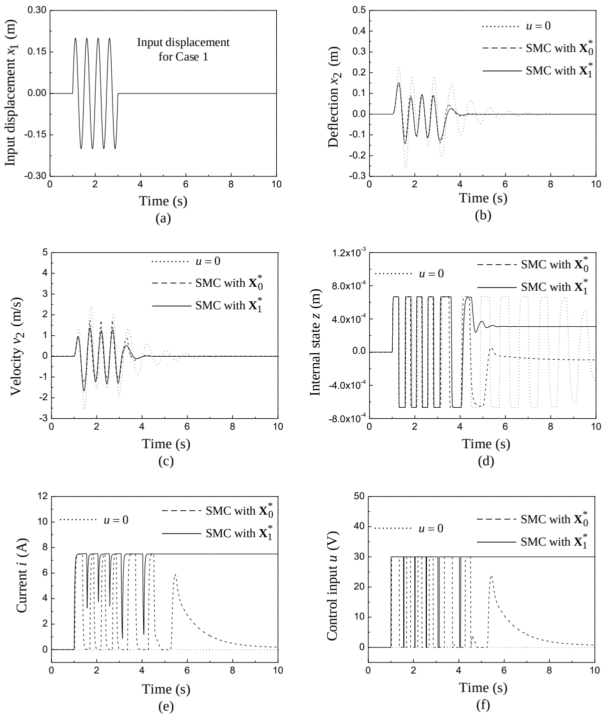

Figure 3 Responses of the proposed SMC for Case 1

proposed SMC are shown in Figs. 3 (b)-(d). One can find that the responses of state by using SMC with $\mathbf{X}_1^*$ rapidly converge toward to the steady state for Case 1. The SMC with $\mathbf{X}_1^*$ produces more control effort than the SMC with $\mathbf{X}_0^*$ to restrain the vibration shown in Figs. 3 (e) and (f). The responses of s and $\ddot{x}_B$ by using SMC with $\mathbf{X}_0^*$ and $\mathbf{X}_1^*$ for Case 1 are shown in Fig. 4. One can find that the responses of s and $\ddot{x}_B$ by using SMC with $\mathbf{X}_1^*$ rapidly converge toward to the steady state. The SMC with $\mathbf{X}_1^*$ seems to has well tracking ability to converge the s

and restrain the acceleration $\ddot{x}_B$ of the vehicle shown in Figs. 4 (a) and (b). It is found that the responses of s keep in the limit range shown in Fig. 5 (a). The SMC with $\mathbf{X}_1^*$ seems to has the well tracking ability to restrain the acceleration $\ddot{x}_B$ of the vehicle shown in Figs. 5 (b).

The responses of s and $\ddot{x}_B$ by using SMC with different $\mathbf{X}^*$ are shown in Fig. 6. It is found that the responses of s keep in the boundary layer shown in Fig. 9 (a). The SMC with $\mathbf{X}_1^*$ seems to has well ability to restrain the acceleration $\ddot{x}_B$ of the vehicle shown in Figs. 9 (b). Finally, the control strategies of $u=0$, SMC with $\mathbf{X}_0^*$ and SMC with $\mathbf{X}_1^*$ have been applied to the suspension system for Cases 1-3. The control object is to minimize the acceleration $\ddot{x}_B$ and the criterion function J of Eq. (26) for Cases 1-3 are illustrated in Table 2. It is found that the SMC with $\mathbf{X}_1^*$ for the three cases has the minimum criterion value among the three control strategies.

4.1.2 Discussion and Summary

The LuGre model is employed to fit for the hysteresis phenomenon of the MR damper in this study. The vibration control strategy is proposed by using SMC with full available outputs. There are three control strategies of $u=0$, SMC with $\mathbf{X}_0^*$ and SMC with $\mathbf{X}_1^*$ have been applied to the suspension system to perform vibration control. From the numerical simulations, it is found that SMC with the $\mathbf{X}_1^*$ shows the well vibration control performance to restrain the acceleration of the vehicle. According to the energy conservation of the suspension system, the i^* in $\mathbf{X}_1^*$ is designed as a variable along with the absolute deflection x_2 of the suspension system. When the external force is applied to drive the deflection x_2, the more damping force should be produced to absorb the external force. From simulation results, the proposed SMC with $\mathbf{X}_1^*$ demonstrates the well vibration control ability for the different kinds of road displacement.

5 Conclusions

The vibration control for the suspension system with MR damper is studied in this paper. The LuGre model is to fit for the hysteresis phenomenon of the MR damper in this study. The dynamic model of the suspension system with MR damper is successfully formulated with the LuGre model and electric circuit, which is controlled by the input voltage. The vibration control strategy is proposed by using SMC with full available outputs. Additionally, the adjustable desired trajectory $\mathbf{X}_1^*$ based on the conservation of energy is proposed. According to the suspension travel, the coefficient of MR damper is adjusted by input current to absorb the external input energy. A criterion function J of the normalized body acceleration for the suspension is proposed to evaluate the vibration control performance. From the numerical simulations, the SMC with $\mathbf{X}_1^*$ has the minimum J for the three cases of tire deflection. The proposed SMC can respectively deal with the full and part available outputs to restrain the acceleration of sprung mass of the suspension system.

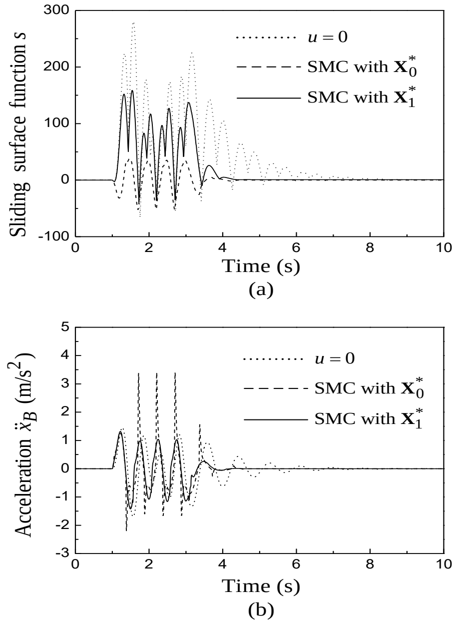

Figure 4 Responses of s and $\ddot{x}_B$ by the proposed SMC for Case 1

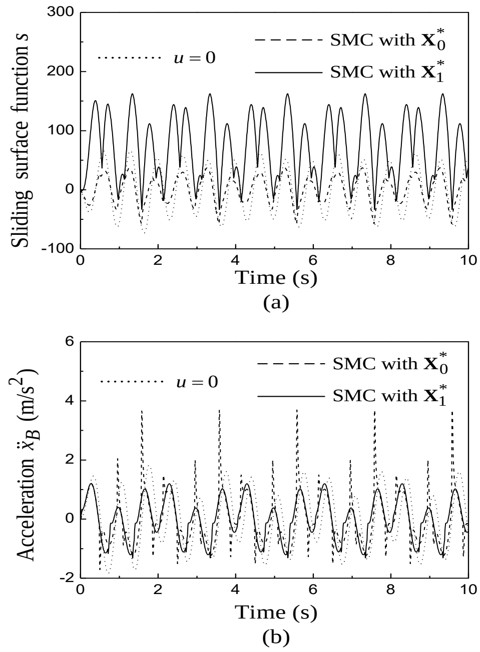

Figure 5 Responses of s and $\ddot{x}_B$ by the proposed SMC for Case 2

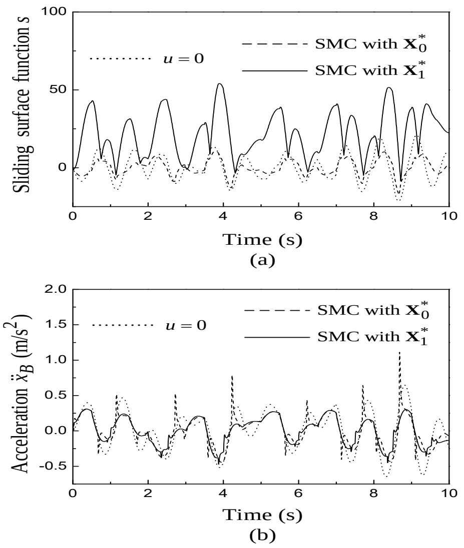

Figure 6 Responses of s and $\ddot{x}_B$ by the proposed SMC for Case 3

Table 2 Criterion value of the three control strategies for Cases 1-3

	Case 1	Case 2	Case 3
$u=0$	$J=0.0471$	$J=0.0994$	$J=0.0295$
SMC with $\mathbf{X}_0^*$	$J=0.0507$	$J=0.0901$	$J=0.0227$
SMC with $\mathbf{X}_1^*$	$J=0.0392$ (Minimum)	$J=0.0718$ (Minimum)	$J=0.0193$ (Minimum)

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