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Response Analysis of A Preloaded Spring Cam System Undergoing An Angular Acceleration

角加速度下預載彈簧凸輪系統之響應分析

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Abstract

The transverse vibration of a translating roller-follower cam with a preloaded spring undergoing an angular acceleration is investigated. A preloaded spring is established to maintain contact between the roller and the cam. The cycloidal displacement is employed to design the rise-dwell-fall-dwell (RDFD) motion of the follower. The deformation of the flexible follower coupled with the rigid-body translation is considered. Hamilton's principle and the assumed mode method are applied to derive the system governing equations of motion. The follower vibration responses undergoing an angular acceleration are obtained using Runge-Kutta method. From the numerical studies, it is shown that the vibration responses are affected significantly by the cam rotational speed. When the rotational speed increases, the vibration response tends to enlarge during the rise, fall and dwell segments after several cycles. When the higher angular acceleration is considered, the response gets large earlier. Under the angular acceleration, the vibration response is larger when the bigger stiffness coefficient value of the preloaded spring is applied.

Keywords: spring, cam, angular acceleration, response analysis

摘要

角加速度下預載彈簧的平移式滾子從動件凸輪的側向振動於本文中研究。設置一預載彈簧用以保持滾子與凸輪的接觸，以擺線輪廓來設計從動件的上昇-停滯-下降-停滯運動，並考慮撓性從動件變形與剛體位移的耦合。應用漢彌頓原理與假設模態法來導出系統統御運動方程式。以阮奇-庫達法求解角加速度下從動件振動響應，數值結果顯示振動響應明顯受凸輪轉速的影響，當轉速增加，在數週後，於上升、下降及停滯段，振動響應趨向增大。當考量較大的角加速度，響應亦比較早變大。角加速度下，使用較大勁度值的預置彈簧，其振動響應會較大。

關鍵字：彈簧，凸輪，角加速度，響應分析

1. INTRODUCTION

Chen [1] introduced and discussed the related topics of cam driven mechanisms. Several studies on the kinematics analysis of the cam mechanisms have been reported [2-3]. Some researchers also studied the dynamics of cam mechanisms. Yousuf [4] analyzed a polydyne cam and a roller follower with different cam rotational speeds. The chaos has been investigated based on positive Lyapunov exponent value. The effect of follower guide's clearance on chaos was considered. The contact between cam and follower was simulated by using Solidworks program. The power spectrum of Fast Fourier Transform and phase plane have been examined the follower non-periodicity during follower motion in the y-direction. Rosenstein program was used to calculate largest Lyapunov exponent. The simulation and experimental results are compared and verified for largest Lyapunov exponent.

Yao etc. [5] proposed a high-static-low-dynamic stiffness (HSLDS) isolator using cam-roller-spring mechanism with a specially designed cam profile. Firstly, the cam design theory was described and a cam was established. The dynamic response of the HSLDS isolator under base excitation was acquired by the Averaging Method. The stability and parameters effect of the dynamic system were discussed. An experimental test system was also set up to verify the isolation performance of the HSLDS isolator and its corresponding linear isolator under base excitation. The proposed HSLDS isolator could achieve a better vibration isolation performance than the linear isolator, especially in the low-frequency range. Chang etc. [6] investigated the transverse vibration of a translating roller-follower cam with a preloaded spring due to the flexible follower rod under constant rotating speed. The effects of some system parameters on the vibration of the flexible follower have been studied.

In this paper, the transverse vibration of a translating roller-follower cam with a preloaded spring undergoing an angular acceleration is studied. A preloaded spring is set to maintain the contact between the roller and the cam during the motion cycle. Lagrange multipliers of two geometric constraints are added to the Hamilton's principle to establish the system governing equations of motion. The assumed mode method is applied to expand the transverse deflection of the follower and Runge-Kutta integration method is applied to calculate the vibration response of the follower.

2. DERIVATION OF GOVERING EQUATIONS

Figure 1 shows a translating roller-follower cam with a preloaded spring. The roller-follower consists of a follower rod that has a separate part, the roller pinned to the follower stem. The cam is assumed to be rigid. Rayleigh beam theory is used to model the flexible follower rod. A preloaded spring is established to maintain the contact between the roller and the cam.

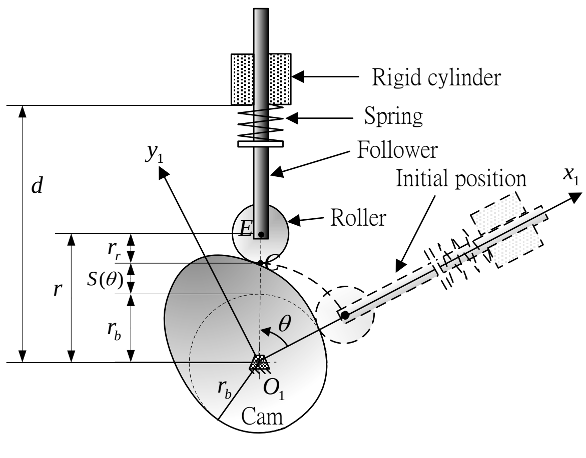

Fig. 1 Schematic of a translating roller-follower

cam with a preloaded spring

Figure 1 shows the schematic of a cam mechanism. The displacement function of the follower rod when the cam rotates with an angle θ is denoted as $S(\theta)$. The same rise-dwell-fall-dwell (RDFD) motion considered in [6] is studied in this paper. The cam profile is considered with rise and fall motions of the cycloidal displacement. The displacement function $S(\theta)$ for the rise segment is given with the following function: (Chen [1])

$$0 \le \theta \le \beta: \ S(\theta) = S_T[\frac{\theta}{\beta} - \frac{1}{2\pi}\sin(\frac{2\pi\theta}{\beta})] \quad (1)$$

where β is the period of the rise segment and S_T is the total lift magnitude. In this study, the period of the rise and fall segment is set to be $\frac{\pi}{2}$. The above motion is also used for the rise portion. To convert rise function to fall function, one subtracts the rise displacement function $S(\theta)$ from the maximum lift S_T.

The cam profile is determined by using the envelope theory. Referring to Fig. 1, one can derive the profile coordinates (x_{1C}, y_{1C}) as

$$x_{1C} = r\cos\theta - \frac{r_r Q}{\sqrt{P^2 + Q^2}}, \quad (2a)$$

$$y_{1C} = r\sin\theta + (x - r\cos\theta)\frac{P}{Q}. \quad (2b)$$

where

$$r = r_b + r_r + S(\theta), \quad (3a)$$
$$P = r\sin\theta - S'(\theta)\cos\theta, \quad (3b)$$
$$Q = r\cos\theta - S'(\theta)\sin\theta. \quad (3c)$$

in which r_b is the base-circle radius of the cam, and r_r is the roller radius.

From Fig. 1, the coordinates (x_{1E}, y_{1E}) of the roller center are observed as

$$x_{1E} = r\cos\theta, \quad (4a)$$
$$y_{1E} = r\sin\theta. \quad (4b)$$

Two fixed frames $O_2 - xy$ and $O_1 - XY$ as shown in Fig. 2 are used. The $O_2 x$ axis coincides with the centerline of the un-deformed rod. The flexible follower undergoes a transverse deflection, $v(x,t)$. The end point E moves to be E' after deformation. The transverse deflection at the end point E are denoted as v_E, i.e., $v_E = v(l,t)$. A rotating frame $O_1 - x_1 y_1$ fixed on the rotating cam is used. The fixed coordinates for the points C and E are

$$X_C = x_{1C}\sin\phi - y_{1C}\cos\phi, \quad (5a)$$
$$Y_C = x_{1C}\cos\phi + y_{1C}\sin\phi, \quad (5b)$$
$$X_E = x_{1E}\sin\phi - y_{1E}\cos\phi, \quad (5c)$$
$$Y_E = x_{1E}\cos\phi + y_{1E}\sin\phi. \quad (5d)$$

where $\phi = \int_0^t \Omega(\tau)\,d\tau$ and Ω is the rotating speed of the cam.

The angular acceleration α in this study is assumed to be constant, so the angular speed of the cam is given as

$$\Omega(t) = \Omega_0 + \alpha t \quad (6)$$

where Ω_0 is the initial angular speed of the cam.

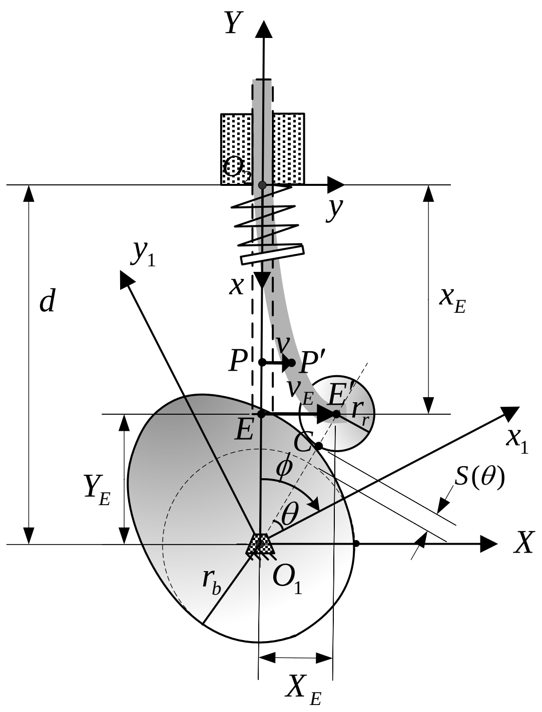

Fig. 2 Deformed configuration of the cam mechanism

As shown in Fig. 2, two constraint equations for the point E are derived from the geometric relationship as

$$\Phi_1 = X_E - v_E = 0 \quad (7)$$

$$\Phi_2 = Y_E + x_E - d = 0 \quad (8)$$

Applying the same approach of Chang etc. [6], one can formulate the kinetic energy and strain energy of the follower, the kinetic energy of the roller, the strain energy of the preloaded spring, and the work done by the constraint forces. The assumed mode method is applied to expand the follower deflections.

$$v(x(t),t) = \sum_{i=2}^{N} b_i(t)x(t)^i \quad (9)$$

where x^i is the mode shape which depends on time since the follower length is varibale. $b_i(t)$ is the associated amplitudes.

The governing equations of the flexible follower rod undergoing an angular acceleration are derived by employing Hamilton's principle.

$$\int_{t_1}^{t_2} \delta(T_{rod} + T_{roller} - U_{rod} + \lambda_1\Phi_1 + \lambda_2\Phi_2)dt = 0 \quad (10)$$

where T_{rod} and T_{roller} are the kinetic energy of the follower rod and the roller, respectively. U_{rod} is the strain energy of the follower rod. $\lambda_1\Phi_1$ and $\lambda_2\Phi_2$ are the works done by the constraint forces.

The derives equation is expressed as

$$\mathbf{M}(\mathbf{Q})\ddot{\mathbf{Q}} + \mathbf{N}(\mathbf{Q},\dot{\mathbf{Q}}) + \mathbf{\Phi}_{\mathbf{Q}}^T\boldsymbol{\lambda} = \mathbf{0} \quad (11)$$

where $\mathbf{M}$, $\mathbf{N}$,and $\boldsymbol{\lambda}$ are mass matrix, nonlinear vector, and Lagrange multiplier, respectively. $\mathbf{Q}$ is the generalized coordinates vector and expressed as

$$\mathbf{Q} = [b_1 \quad b_2 \quad \cdots \quad b_N \quad x_E \quad \theta]^T. \quad (12)$$

The two constraints as expressed in equations (7) and (8) are combined as the following form

$$\mathbf{\Phi}(\mathbf{Q}) = [\Phi_1 \quad \Phi_2]^T = \mathbf{0} \quad (13)$$

Using the partitioning method (Parviz [7]), one can derive the second order nonlinear ordinary differential equation in independent generalized coordinate. The vibration response of the follower with a preloaded spring undergoing an angular acceleration can be solved from the equations by applying Runge-Kutta integration method.

3. NUMERICAL RESULTS AND DISCUSSIONS

An example is given to investigate the vibration of the translating roller-follower cam with a preloaded spring undergoing an angular acceleration for RDFD case. The cycloidal displacement motion is applied to model the rise and fall displacement curve. The total rise S_T is set to 15 mm. The period of the rise and fall segment β is set to be $\frac{\pi}{2}$. The stiffness coefficient value of the preloaded spring is given as $k_s = 1.26 \times 10^2$ kg/s^2 (or Nt/m). The initial amount of compression of the spring is $x_o = 3.6$ mm. The elastic modulus of the follower rod is $E = 2.1 \times 10^8$ kg/mm $\cdot$ s^2. And its density is $\rho = 7.8 \times 10^{-6}$ kg/mm^3. The cross section of the follower rod is a circle with radius of $r_f = 5$ mm. The associated cross-sectional area and area inertia are $A = 78.54$ mm^2 and $I = 490.87$ mm^4. The distance from the lower end of the rigid cylinder and the rotation center of the cam is $d = 112$ mm. The base-circle radius of the cam is $r_b = 26$ mm. The radius, mass, and mass polar moments of inertia of the roller are $r_r = 5$ mm, and $m_r = 0.05$ kg, respectively.

In the previous study ([6]), the numerical results nearly converge with $N = 3$, so the assumed mode method with $N = 3$ is applied in this study. The vibration response of the swinging roller-follower cam with an angular acceleration is investigated. Four cases of angular accelerations including 0, 9, 18, and 36 rad/s^2 are studied. The initial angular speed is given as 260 *rad/s*. The transverse vibration responses of the output node for seven cam cycles are plotted. They are shown in Figures 3-6. Figure 3 shows the vibration response curve of the translating follower cam at constant angular speed of 260 *rad/s*. The amplitude of the vibration response of the follower is about the same order during the rise and fall segments while the amplitude is very small during the dwell intervals.

Figure 4 shows the vibration response of the system with $\Omega_0 = 260\, rad/s$ undergoing the angular acceleration of 9 rad/s^2. The response is almost the same as that for constant angular speed. When the angular acceleration gets to be 18 rad/s^2, the response shown in Fig. 5 at the rise and fall segments still remain similar to that for $\alpha = 9\ rad/s^2$. However, in the dwell interval, the follower vibration response increases obviously about after 6 cycles. Figure 6 shows the vibration response of the system with $\Omega_0 = 260\, rad/s$ undergoing the angular acceleration of 36 rad/s^2. It is found that the response amplitude at the rise and fall segments enlarges as the angular speed increases after 6 cycles. The response at the dwell segments enlarges as the angular speed increases after 4 cycles. The response gets large faster as the cam undergoing a larger angular acceleration. The response gets large earlier, when the higher angular acceleration is applied.

To investigate the influences of the stiffness of the preloaded spring on the response, two different spring stiffness values, $k_s = 1.26 \times 10^3$ and 3.96×10^3 kg/s^2, are given and their associated responses are also discussed. Figure 7 shows the response of the cam system with $\Omega_0 = 260\, rad/s$ undergoing the angular acceleration of 36 rad/s^2 and the preloaded spring stiffness $k_s = 1.26 \times 10^3$ kg/s^2. It is found that the response gets larger when the rotating angle gets larger. The response amplitude is larger a little bit than that for the preloaded spring stiffness $k_s = 1.26 \times 10^2$ kg/s^2. When a large preloaded spring stiffness $k_s = 3.96 \times 10^3$ kg/s^2 is applied, the response gets larger obviously than those for the preloaded spring stiffness $k_s = 1.26\times10^2$ kg/s^2 and $k_s = 1.26\times10^3$ kg/s^2. From the numerical results, it is shown that the response of the follower end undergoing the angular acceleration is larger when the bigger stiffness coefficient value of the preloaded spring is considered.

4. CONCLUSIONS

The equations of motion for the vibration of a translating roller-follower cam with a preloaded spring for RDFD case are derived by using Hamilton's principle and the assumed mode method. The flexibility of the follower rod is considered and modeled as a Rayleigh beam. A preloaded spring is set to maintain the contact between the roller and the cam. Two geometric constraints are formulated to be added to the Hamilton's principle with Lagrange multipliers.

From the numerical studies, it is shown that the vibration responses are affected significantly by the cam rotational speed. When the rotational speed increases, the vibration response tends to enlarge during the rise, fall and dwell segments after several cycles. When the higher angular acceleration is considered, the response gets large earlier. Under the angular acceleration, the vibration response is larger when the bigger stiffness coefficient value of the preloaded spring is applied.

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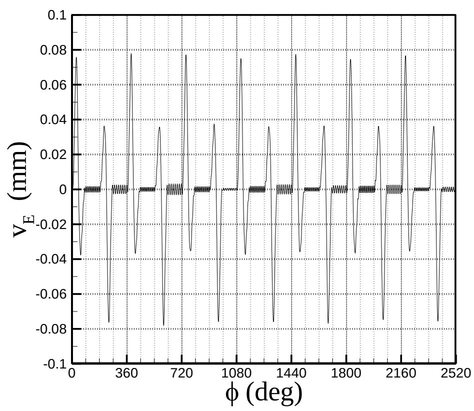

Figure 3 The vibration response v_E with a preloaded spring $k_s = 1.26\times10^2$ kg/s^2 and a constant angular speed $\Omega = 260\, rad/s$.

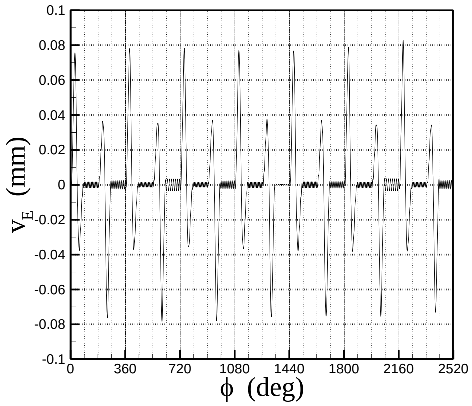

Figure 4 The vibration response v_E with a preloaded spring $k_s = 1.26 \times 10^2$ kg/s^2 and $\Omega_0 = 260\, rad/s$ and $\alpha = 9\ rad/s^2$.

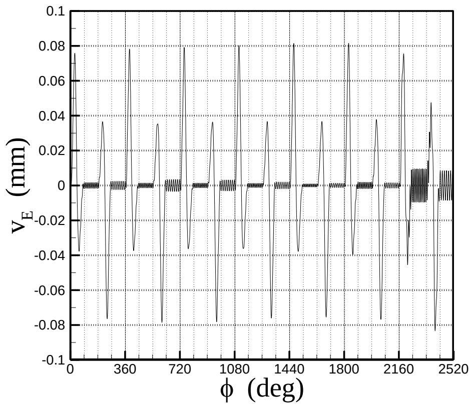

Figure 5 The vibration response v_E with a preloaded spring $k_s = 1.26 \times 10^2$ kg/s^2 and $\Omega_0 = 260\, rad/s$ and $\alpha = 18\ rad/s^2$.

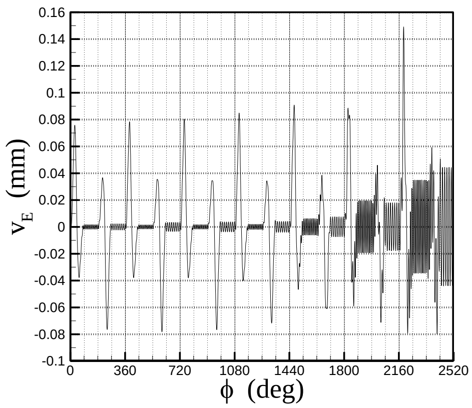

Figure 6 The vibration response v_E with a preloaded spring $k_s = 1.26 \times 10^2$ kg/s^2 and $\Omega_0 = 260\, rad/s$ and $\alpha = 36\ rad/s^2$.

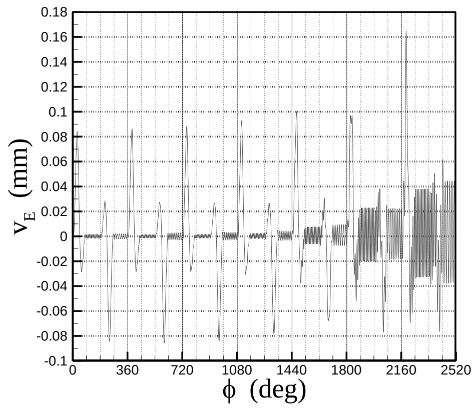

Figure 7 The vibration response v_E with a preloaded spring $k_s = 1.26 \times 10^3$ kg/s^2 and $\Omega_0 = 260\, rad/s$ and $\alpha = 36\ rad/s^2$.

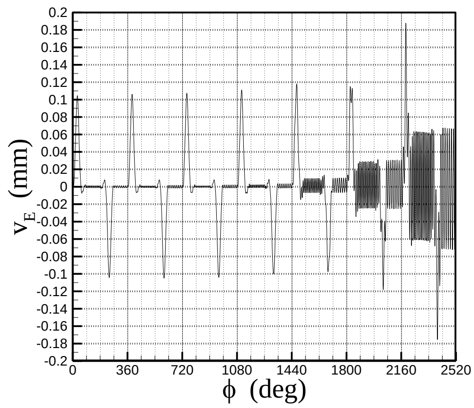

Figure 8 The vibration response v_E with a preloaded spring $k_s = 3.96 \times 10^3$ kg/s^2 and $\Omega_0 = 260\, rad/s$ and $\alpha = 36\ rad/s^2$.