

The effect of non-linear Boussinesq and internal heat source on free convection over a vertical permeable plate of uniform flow in porous media

在多孔性介質內具非線性布辛克相似解及內部熱源對於自由對流流經一均勻流體垂直可穿透平板的效果

Ken-Ming Tu^{1,*} Kuo-Ann Yih² Chuo-Jeng Huang³ Yi-Wei Chen⁴
杜耿銘¹ 易國安² 黃卓初³ 陳一葳⁴

^{1,3}Department of Aircraft Engineering, Air Force Institute of Technology

^{1,2,3,4} 空軍航空技術學院飛機工程系

Abstract

The numeral solution is represented for the effects of non-linear Boussinesq and internal heat source on free convection over a vertical permeable plate in porous media. The surface of the vertical permeable plate is maintained at variable wall temperature. The internal heat source coefficient is of an exponential form. The partial differential equation is transformed into a non-similar equation and then solved by Keller box method (KBM). Compared with the previously published articles, we obtain the very good consistency. Numerical data for the dimensionless temperature profile, the dimensionless velocity profile and the local Nusselt number are presented in graphical and tabular forms for the exponent of wall temperature λ , the blowing/suction parameter ξ , the internal heat source coefficient A^* , the non-linear temperature parameter δ . Discuss physical issues in detail. The increase in the nonlinear temperature parameter δ indicates an increase in the local Nusselt number.

Keywords : Non-linear Boussinesq, Internal heat source, Natural convection, Vertical permeable plate, Porous medium.

摘要

在多孔性介質中具非線性布辛克和內部熱源對於自由對流流經均勻的垂直平板之影響。垂直可穿透板的表面保持均勻的壁溫。內部熱源的形式假設為指數漸減形式。將偏微分方程轉換為類似方程，然後以凱勒盒子法求解。所得的數值結果與以前發表的文章相比，我們獲得了非常好的一致性。本文數值計算的結果主要以圖形與表格的形式來顯示：壁溫的指數型式 λ ，噴/吸流參數 ξ ，內部熱源係數 A^* ，非線性溫度參數 δ 的數據。文中的物理現象也詳加討論。非線性溫度參數 δ 的增加表示局部努塞爾數的增加。

關鍵字：非線性布辛克，內部熱源，自然對流，垂直可穿透平板，多孔性介質

1. Introduction

Saturated porous medium in the convective heat and mass transfer in geothermal and geophysical engineering has many important applications. These include nuclear reactor cooling systems, geothermal energy extraction, building insulation, filtration technology and the underground disposal of nuclear waste. The recent work of Nield and Bejan [1], Ingham and Pop [2] and Vafai [3] provides a comprehensive overview of the information available in this field.

In pure heat transfer, change the body separating any shape. A new class of similarity solutions has obtained for isothermal vertical plate in a semi-infinite quiescent fluid with internal heat source coefficient decaying exponentially by Crepeau and Clarksean [4]. Postelnicu and Pop [5] studied the similarity solutions of free convection boundary layers over vertical and horizontal surfaces in porous media with internal heat source coefficient. Postelnicu et al. [6] presented the case of the vertical permeable flat plate. Bagai [7] analyzes the similar solutions of arbitrary convective boundary layers in porous media with internal heat source coefficient. Mohamed [8] carried out a similar solution to the effect of transverse mass flux on the natural convective boundary layer caused by heating the vertical plate embedded in a saturated porous medium with internal heat source coefficient. Yih et al. [9] examined uniform blowing/suction and Soret/Dufour effects on heat and mass transfer by natural convection about a vertical flat plate in porous media: UHF/UMF.

Assume that the internal heat source coefficient is the exponential decay form. The control equations are solved using the Keller Box method (KBM). The

results of the various parameter values were obtained and discussed.

2. Analysis

Let us consider the problem of the effect of non-linear Boussinesq and internal heat source on natural convection over a vertical permeable plate in a porous medium. Figure 1 shows the flow model and physical coordinate system. The origin of the coordinate system is placed at the origin of the vertical permeable plate, where x is the coordinate along the surface of plate measured from the origin and y is the coordinate normal to the surface, respectively. We consider the boundary condition of variable wall temperature T_w (VWT); T_w are higher than the ambient temperature T_∞ .

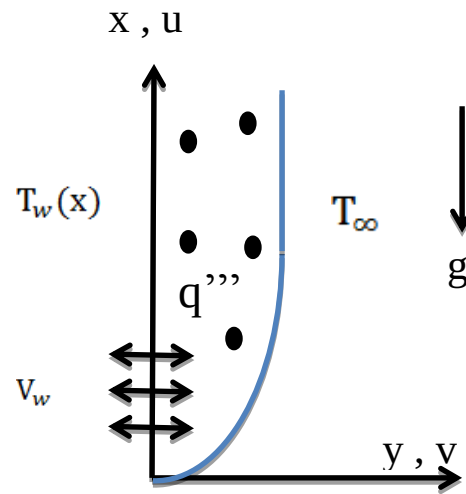


Fig. 1. Flow model and the physical coordinate system

Use the boundary layer and Boussinesq approximations, the governing equations and the boundary conditions based on the Darcy law can be written as follows:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

Momentum (Darcy) equation:

$$u = -\left(\frac{K}{\mu}\right)\left[\left(\frac{\partial p}{\partial x}\right) + \rho g\right] \quad (2)$$

$$v = -\left(\frac{K}{\mu}\right)\frac{\partial p}{\partial y} \quad (3)$$

Energy equation:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{q'''}{\rho_\infty C_p}, \quad (4)$$

Boussinesq approximation:

$$\rho = \rho_\infty \left[1 - \beta_0 (T - T_\infty) - \beta_1 (T - T_\infty)^2 \right], \quad (5)$$

Boundary conditions:

$$y = 0: v = V_w, \quad T = T_w(x) = T_\infty + Ax^\lambda \quad (6.1-3)$$

$$y \rightarrow \infty: u = 0, \quad T = T_\infty \quad (7.1-3)$$

Here, u and v are the Darcian velocities in the x - and y - directions; g is the gravitational acceleration; p and ρ are the pressure and the density of the fluid, respectively; q''' is the internal heat source coefficient rate per unit volume; C_p is the specific heat at constant pressure; β_0 and β_1 are the thermal expansion coefficients of the fluid; λ is the exponent of VWT.

The article assumes that the boundary layer is very thin relative to the main vertical plate.

We now pay attention to the governing equations (2) and (3). If we do the

cross-differentiation $\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$ then the pressure terms

in equations (2) and (3) can be eliminated. Further, with the help of the equation (5) and the boundary layer approximation ($\partial/\partial x \ll \partial/\partial y$), then we can obtain

$$\frac{\partial u}{\partial y} = \left(\frac{\rho_\infty g \beta_0 K}{\mu} \right) \left[\frac{(T_w - T_\infty) Rax^{1/2}}{x} \theta' \right] \quad (8)$$

Integral equation (8) once, with boundary conditions (7.1-3), and then get

$$u = \frac{\rho_\infty g \beta_0 K}{\mu} [T - T_\infty] \quad (9)$$

Invoking the following dimensionless non-similarity variables:

$$\xi = \frac{2V_w x}{\alpha Ra_x^{1/2}} \quad (10.1)$$

$$\eta = \frac{y}{x} Ra_x^{1/2} \quad (10.2)$$

$$f(\xi, \eta) = \frac{\psi}{\alpha Ra_x^{1/2}} \quad (10.3)$$

$$\theta(\xi, \eta) = \frac{T - T_\infty}{T_w(x) - T_\infty} \quad (10.4)$$

$$Ra_x = \frac{\rho_\infty g \beta_0 K (T_w(x) - T_\infty) x}{\mu \alpha} \quad (10.5)$$

The internal heat source coefficient rate per unit volume q''' is modeled according to the following equation.

$$q''' = A^* \frac{k(T_w(x) - T_\infty)}{x^2} Ra_x e^{-\eta} \quad (11)$$

Here, k is the equivalent thermal conductivity. A^* is the internal heat source coefficient.

Note that when $A^* = 0$ corresponds to the case 1: no internal heat source coefficient (designated as NIHG); while for $A^* > 0$ corresponds to the case 2: with internal heat source coefficient (WIHG).

Substituting equations (10)-(11) into equations (9), (4) - (7), we obtain

$$f' = u + \frac{\delta}{2} u^2 \quad (12)$$

$$\theta'' + \frac{1+\lambda}{2} f\theta' - \lambda f'\theta + A^* e^{-\eta} = \frac{1-\lambda}{2} \xi \left(f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi} \right) \quad (13)$$

The boundary conditions are defined as follows:

$$\eta = 0: f = -\frac{\xi}{2}, \theta = 1 \quad (14)$$

$$\eta \rightarrow \infty: \theta = 0 \quad (15)$$

In addition, in terms of the new variables, the Darcian velocities in x - and y - directions are, respectively, given by

$$u = \frac{\alpha Rax}{x} f' \quad (16)$$

$$v = -\frac{\alpha Ra_x^{1/2}}{2x} \left[(1+\lambda) f + (1-\lambda) \left(\frac{\partial f}{\partial \xi} \xi - f' \eta \right) \right] \quad (17)$$

where primes denote differentiation with respect to η .

The results of practical interest in many applications is the surface heat transfer rate. The surface heat transfer rate is expressed in terms of the local Nusselt number Nu_x , which is basically defined as follows:

$$Nu_x = \frac{h_x x}{k} = \frac{q_w x}{[T_w - T_\infty] k} \quad (18)$$

where h_x is local convective heat transfer coefficient; q_w are the local heat flux; and $q_w = h_x \cdot [T_w - T_\infty]$ (the Newton's law of cooling).

From the Fourier's law of heat conduction, the rate of surface heat transfer q_w is defined as follows:

$$q_w = -k \left(\frac{\partial T}{\partial y} \right) \bigg|_{y=0} \quad (19)$$

Inserting equations (19) into equations (18) and with the aid of equation (10), the local Nusselt number Nu_x in terms of $Ra_x^{1/2}$ is obtained by

$$\frac{Nu_x}{Ra_x^{1/2}} = -\theta'(\xi, 0) \quad (20)$$

3. Numerical Method

The current analysis of the integrated system equation (12) - (15) implicit finite difference approximation is used to modify the Keller box Cebeci and Bradshaw [10]. First, the partial

differential equation is transformed into three first-order equations. Then, these first-order equations are expressed in finite difference form and solved by the iterative scheme along with their boundary conditions. This approach gives a better rate of convergence and reduces the numerical computational times.

Computations were carried out on a personal computer with the first step size $\Delta\eta_1 = 0.01$. The variable grid parameter is chosen 1.01 and the value of $\eta_\infty = 15$. The iterative procedure is stopped to give the final temperature distribution when the errors in computing the $|\theta'_w|$ in the next procedure become less than 10^{-8} .

4. Results and Discussion

In order to verify the accuracy of our present method, we have compared our results with those of Cheng [11], Yih [12], Huang [13]. Table 1 shows the comparison of the values of $-\theta'(\xi, 0)$ for various values of ξ with $A^* = 0$ (NIHG), $\lambda = 0$, $\delta = 0$. Table 2 lists the comparison of the values of $-\theta'(0, 0)$ for various values of n and λ with $A^* = 1$ (WIHG), $\delta = 0$, $\xi = 0$. Table 3 shows the comparison of the values of $-\theta'(\xi, 0)$ for various values of ξ with $A^* = 0$, $\delta = 0$, $\lambda = 0$. The comparisons in all the above cases are found to be in excellent agreement, as shown in Tables 1-3.

Table 1 Comparison of the values of $-\theta'(\xi, 0)$ for various values of ξ with $A^* = 0$ (NIHG), $\lambda = 0$, $\delta = 0$

ξ	Yih[12]	Cheng [11]	Present results
	$\lambda = 0$	$\lambda = 0$	$\lambda = 0$
-10	4.9999	—	4.9981
-4	2.0015	—	2.0011

0	0.4437	0.4439	0.4437
4	0.0333	—	0.0329
10	0.0001	—	0.0000

Table 2 Comparison of the values of $-\theta'(0,0)$ for various values of λ with $A^* = 1$ (WIHG), $\delta = 0$, $\xi = 0$

Yih [12]		Present results	
$\lambda = 0$	$\lambda = 1$	$\lambda = 0$	$\lambda = 1$
-0.2153	0.5241	-0.2153	0.5241

Table 3 Comparison of the values of $-\theta'(\xi,0)$ for various values of ξ with $A^* = 0$, $\delta = 0$, $\lambda = 0$.

ξ	Huang [13]	Present results
-4	2.0014	2.0011
-2	1.0725	1.0725
0	0.4437	0.4437
2	0.1407	0.1411
4	0.0329	0.0329

The numerical results are presented for the internal heat source coefficient coefficient A^* ranging from 0 to 1, the exponent of wall temperature λ ranging from 0 to 1.0, the non-linear temperature parameter δ ranging from 0 to 1, the blowing/suction parameter ξ ranging from -10 to 10.

The effects of the non-linear temperature parameter δ on the dimensionless velocity and temperature profiles with $\lambda = 0$, $\xi = 0$, $A^* = 0$ are illustrated in Figs. 2 and 3, respectively. In Fig. 2, for $\delta = 0$ and $\delta = 1$, the dimensionless velocity profile obtained by them are equal. Only $\delta = 2$, the dimensionless velocity profile increases significantly. In Figure 3, the increase in the non-linear temperature parameter δ results in a decrease in the

dimensionless temperature profile.

The effects of the non-linear temperature parameter δ on the dimensionless velocity and temperature profiles with $\lambda = 0$, $\xi = 0$, $A^* = 1$ are illustrated in Figs. 4 and 5, respectively. In Fig. 4, in the case of an internal heat source coefficient ($A^* = 1$), the δ value increases and the dimensionless velocity increases. In Figure 5, When $A = 1$, $\delta = 0$, the dimensionless temperature profile of the initial section of a significant increase in the phenomenon.

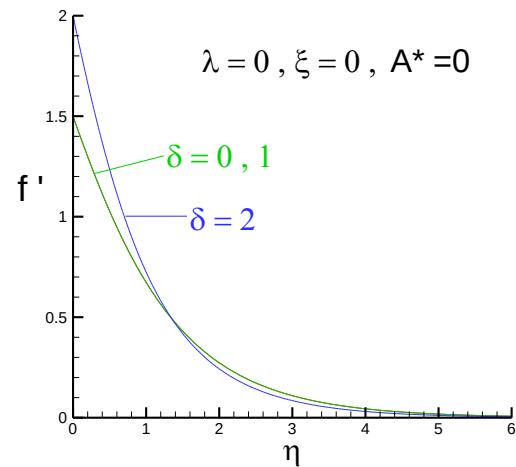


Figure 2 Dimensionless velocity profiles

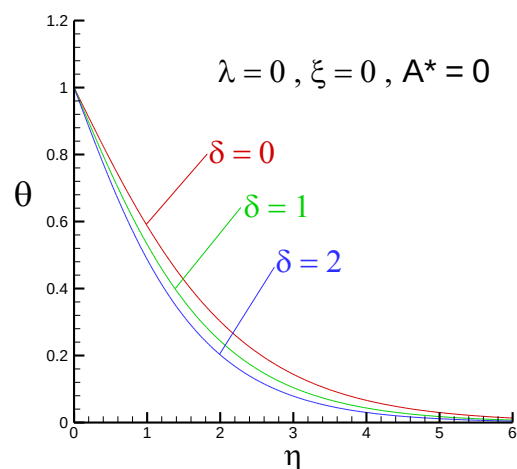


Figure 3 Dimensionless temperature profiles

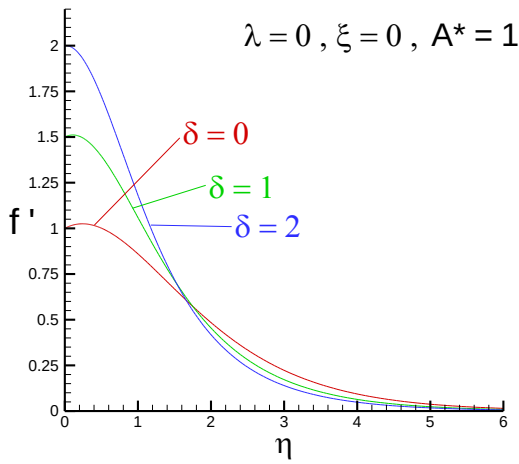


Figure 4 Dimensionless velocity profiles

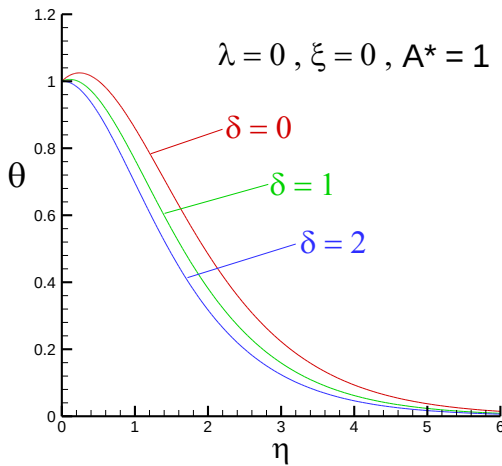


Figure 5 Dimensionless temperature profiles

5. Conclusions

A two-dimensional laminar boundary layer analysis is presented to study the effect of the internal heat source coefficient on the natural convection flow in a Darcy porous medium adjacent to the vertical permeable plate maintained at uniform wall temperature (UWT). The internal heat source coefficient is of an exponential decaying form. The control equations are solved using the Keller Box method (KBM). The results of the various parameter values were obtained and discussed. Comparisons with previously published work are performed and excellent agreement is obtained. Numerical solutions

are obtained for different values of the exponent of wall temperature λ , the blowing/suction parameter ξ , the internal heat source coefficient A^* , the non-linear temperature parameter δ . For example: in figs 3 and 5, When the internal heat source coefficient A^* increases from 0 to 1, the local Nusselt number is diminished. As the non-linear temperature parameter δ enhances from 0 (linear) to 2 (non-linear), the local Nusselt number increases. It can be seen from Fig. 5 that the phenomenon of the heat transfer from the porous medium to the vertical plate is very significant for the case of $\delta = 0$ (linear temperature parameter).

Nomenclature

A^* internal heat source coefficient
 f dimensionless stream function
 g gravitational acceleration
 h_x local convective heat transfer coefficient
 K permeability of the porous medium

Nu_x local Nusselt number

p pressure
 q_w local heat flux

Ra_x modified local Rayleigh number

T temperature
 P pressure
 u Darcy velocity in the x-direction
 v Darcy velocity in the y-direction

V_w variable blowing/suction velocity

x streamwise coordinate
 y transverse coordinate

Greek symbols

α equivalent thermal diffusivity

β_0	coefficient of thermal expansion
β_1	coefficient of thermal expansion
δ	the non-linear temperature parameter
η	pseudo-similarity variable
θ	dimensionless temperature
λ	exponent of wall temperature
ξ	the blowing/suction parameter
ρ	density
ψ	stream function

References

1. Nield, D. A., and Bejan, A., Convection in Porous Media, 4th edn. Springer, New York, 2013.
2. D. B. Ingham and I. Pop, Transport Phenomena in Porous Media III, Elsevier, Oxford, 2005.
3. Vafai, K., Handbook of Porous Media, 2nd edn. Mercel Dekker, New York, 2005.
4. Crepeau, J. C., and Clarksean, R., "Similarity solutions of natural convection with internal heat source coefficient," J. Heat Transfer, Vol. 119, pp. 183-185 (1997).
5. Postelnicu, A., and Pop, I., "Similarity solutions of free convection boundary layers over vertical and horizontal surfaces in porous media with internal heat source coefficient," Int. Commun. Heat Mass Transfer, Vol. 26, pp. 1183-1191 (1999).
6. Postelnicu, A., Grosan, T., and Pop, I., "Free convection boundary-layer over a vertical permeable flat plate in a porous medium with internal heat source coefficient," Int. Commun. Heat Mass Transfer, Vol. 27, pp. 729-738 (2000).
7. Bagai, S., "Similarity solutions of free convection boundary layers over a body of arbitrary shape in a porous medium with internal heat source coefficient," Int. Commun. Heat Mass Transfer, Vol. 30, pp. 997-1003 (2003).
8. Mohamed, E. A., "The effect of lateral mass flux on the natural convection boundary layers induced by a heated vertical plate embedded in a saturated porous medium with internal heat generation," Int. J. Thermal Sci., Vol. 46, pp. 157-163 (2007).
9. Yih, K. A, Huang, C. J., Pan, U. J., Sheu, M. S. and Wang, J. J., "Uniform blowing/suction and Soret/Dufour effects on heat and mass transfer by natural convection about a vertical flat plate in porous media: UHF/UMF," Journal of Air Force Institute Technology, Vol. 13, pp. 29-42 (2014).
10. Cebeci, T., and Bradshaw, P., Physical and Computational Aspects of Convective Heat Transfer, Springer-Verlag, New York, 1984.
11. Cheng, C. Y., "Natural convection heat and mass transfer from a vertical truncated cone in a porous medium saturated with a non-Newtonian fluid with variable wall temperature and concentration," Int. Commun. Heat Mass Transfer, Vol. 36, pp. 585-589 (2009).
12. Yih, K. A., "The effect of uniform lateral mass flux on free convection about a vertical cone embedded in a saturated porous medium," International Communications in Heat and Mass Transfer, Vol. 24, pp. 1195-1205 (1997).
13. Huang, C. J., "Influence of uniform blowing/suction on the free convection of non-Newtonian fluids over a vertical plate in porous media with internal heat source coefficient and Soret/Dufour effects," International Journal of Scientific and Engineering Research (IJSER), Vol. 7, No. 6, pp. 853-869 (2016).

