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Linearized Soliton Solutions of the Equal Width equation

EW方程式的可線性化孤波解

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Abstract

This paper obtains two new solitary wave solutions of the equal width (EW) equation. This partial differential equation with a nonlinear term coefficient is derived by the simplest equation method with the Bernoulli equation as the simplest equation. It is shown that the proposed exact solutions overcome the long existing problem of discontinuity and present a new phenomenon, namely, soliton sliding.

Keywords: equal width equation; the simplest equation method; Bernoulli; soliton sliding.

摘　要

本文利用最簡單法搭配柏努利方程式求解 equal width (EW)方程式，最後得到兩組新的孤波解。新解克服了存在已久的精確解不連續現象，並且呈現出新的物理現象，暫命名為”孤波滑移”。

關鍵字： equal width (EW)方程式， 最簡單法， 柏努利， 孤波滑移

1. Introduction

The study of the nonlinear differential equations has been going on for the past few decades. A large number of equations have been studied in this context. One of the major questions that arises in the study of the differential equations is integrable. In particular, the thirst is to find soliton solutions of such equations. There are many newly developed methods to solve the newly generated nonlinear equations in generalized forms [1-5]. These newly developed methods are a real blessing in this area of Applied Mathematics. In this paper there is one such important equation that will be studied. It is called the equal width (EW) equation. There are various kinds of solution that are already available for this equation. In the following, the focus is on obtaining linearized solutions of the EW equation. The technique that will be used is the simplest equation method with the Bernoulli equation as the simplest equation [6]. The solutions will also be used to illustrate the soliton.

2. Mathematical analysis

The dimensionless form of the EW equation is given by [7]

$$u_t + \alpha u u_x - u_{xxt} = 0 , \qquad (2.1)$$

where α is a positive parameter, as an equally valid and accurate model for the same wave phenomena simulated by the KdV equation. It is soliton solution is [8]

$$u=\frac{3c}{\alpha}\sec h^2[\frac{1}{2}(x-ct)]. \qquad (2.2)$$

This equation called the equal width equation is because the solutions for solitary waves with a permanent form and speed for a given coefficient value of u_{xxt}, are waves with an equal width or wavelength for all wave amplitudes. The EW equation is a special case of the generalized regularized long-wave (GRLW) equation [7-10].

From equation (2.1), using the notation

$$\xi=x-ct, \qquad (2.3)$$

it is possible to get

$$cu_\xi+\alpha uu_\xi-cu_{\xi\xi\xi}=0. \qquad (2.4)$$

The assumed solution for equation (2.4) is taken to be

$$u(\xi)=\sum_{i=0}^{L}a_i[Y(\xi)]^i, \qquad (2.5)$$

where a_i are constants to be determined; Y is the exact solution of a chosen nonlinear ordinary differential equation, called the simplest equation; and, L is a constant to be determined and the power of the specified solution function finite series, Y.

After substituting equation (2.5) into equation (2.4) and balancing the linear term of the derivative's highest order with the highest order nonlinear term, the balance equation gives $L=2$. Therefore, the assumed solution can be constructed as

$$u(\xi)=a_0+a_1Y+a_2Y^2. \qquad (2.6)$$

Then consider the Bernoulli equation as the simplest equation in the form

$$Y_\xi=aY+bY^2, \qquad (2.7)$$

where a,b are constants. The exact solution of the equation (2.7) is [11]

$$Y=\frac{a}{e^{-a(\xi+\xi_0)}-b}, \qquad (2.8)$$

where ξ_0 is an integral constant, here setting ξ_0=0.

Substituting equations (2.6-7) into (2.4), and equating the coefficients of the same powers of Y to zero leads to a system of nonlinear relationships among the parameters of the solution and the parameters of the solved class of equation.

$$Y^0:-ca_0+\frac{\alpha}{2}a_0^2=0, \qquad (2.9)$$

$$Y^1:-ca_1+\alpha a_0a_1+ca_1a^2=0, \qquad (2.10)$$

$$Y^2:-ca_2+\alpha a_0a_2+\frac{\alpha}{2}a_1^2+3ca_1ab+4ca_2a^2=0$$

(2.11)

$$Y^3:\alpha a_1a_2+2ca_1b^2+10ca_2ab=0, \qquad (2.12)$$

$$Y^4:\frac{\alpha}{2}a_2^2+6ca_2b^2=0. \qquad (2.13)$$

Solving equations (2.9-13) yields the following 4 cases.

Case1.

$$a=1,b=\alpha,a_0=0,a_1=-12c,a_2=-12c\alpha, \qquad (2.14)$$

Case2.

$$a=-1,b=\alpha,a_0=0,a_1=12c,a_2=-12c\alpha, \qquad (2.15)$$

Case3.

$$a=1,b=-\alpha,a_0=0,a_1=12c,a_2=-12c\alpha, \qquad (2.16)$$

Case4.

$$a=-1, b=-\alpha, a_0=0, a_1=-12c, a_2=-12c\alpha \tag{2.17}$$

They, therefore, could be derived to new solutions of equation (2.1) as

$$u_1=\frac{-12ce^{-(x-ct)}}{(e^{-(x-ct)}-\alpha)^2}, \tag{2.18}$$

$$u_2=\frac{-12ce^{(x-ct)}}{(e^{(x-ct)}-\alpha)^2}, \tag{2.19}$$

$$u_3=\frac{12ce^{-(x-ct)}}{(e^{-(x-ct)}+\alpha)^2}, \tag{2.20}$$

$$u_4=\frac{12ce^{(x-ct)}}{(e^{(x-ct)}+\alpha)^2}. \tag{2.21}$$

3. Numerical results and discussions

A soliton can be defined as a solution to a nonlinear partial differential equation that exhibits the following three properties [1]: (i) the solution should demonstrate a permanent form wave; (ii) the solution is localized, which means that the solution either decays exponentially to zero, or converges to a constant at infinity; and, (iii) the soliton interacts with other solitons preserving its character.

Therefore, after numerical calculation using the software MATLAB, equations (2.20-21) are soliton solutions, and equations (2.18-19) are travelling wave solutions. In addition, for the solution continuity, equations (2.18-21) are reducible to linear solutions as the nonlinear term α in equation (2.1) approaches zero. Due to the feature of linearity, we observe the soliton solution sliding, which was termed the soliton-sliding phenomenon, as shown in Figure 1.

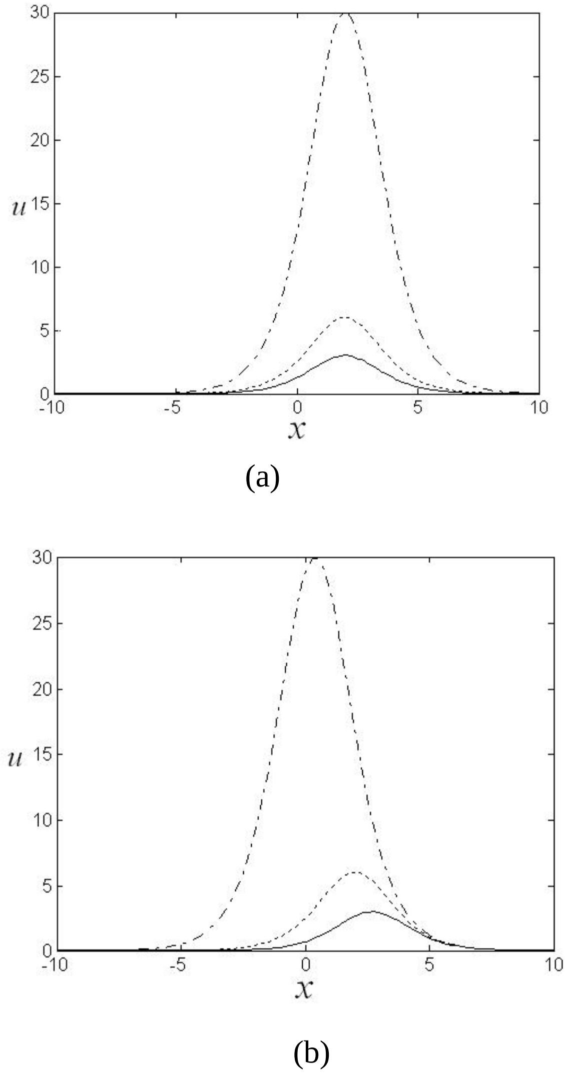

Fig. 1. Influence of the nonlinear term coefficient α. [c = 2, t = 1; (a) equation (2.2), (b) equation (2.21); ——: $\alpha=2$; – – –: $\alpha=1$; –·–: $\alpha=0.2$.]

4. Conclusions

In this paper, the linearized soliton solutions of the EW equation are obtained. Although, the steep slope in the solution (2.2) is maintained at the same location, under the same physical conditions, the new exact solution will not become infinite as α approaches zero. Moreover, the location of the steep slope in the new exact solution slides, which was termed the soliton-sliding phenomenon. The simplest equation method is used to carry out the soliton solutions of this equation. The linearized soliton solutions are elaborated. The reason the new soliton solution slides is entirely due to the influence of linearity. Extensions of the simplest equation method to study integrable nonlinear partial differential equations without linearized solutions are expected in future works.

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