Using neural network and orthogonal array for the application of fire-control data of artillery

Yi-Wei Chen*, Ken-Ming Tu, Chuo-Jeng Huang
Department of Aircraft Engineering, Air Force Institute of Technology, Taiwan R.O.C.

*Corresponding author. E-mail address: chenyiwei530@yahoo.com.tw. Tel: +886-7-6254168

Abstract

The firing precision of artillery is influenced by the controllable and uncontrollable factors, and it will impact the war effectiveness. The fire-control data of artillery are corrected according to the range tables in various environment conditions to obtain better firing precision. The fabrication of traditional man-made method for range tables costs huge amount of ammunition and time, and the modern range tables are based on the precise mathematical ballistic model. In this paper, the algorithm of neural network and orthogonal array method is utilized to build the artillery ballistic model to predict the artillery range. The neural network is capable of building the artillery ballistic model without the complex mathematical model and the utilization of orthogonal array can reduce the requirement for firing data. The proposed method is compared with the traditional regression analysis method and the neural network without using the orthogonal array. The result shows that the proposed method has the better prediction precision and the mean absolute percentage error can approach to 1.422%. This paper concentrates on the utilization of neural network and orthogonal array to build the complex artillery ballistic model and offers a usable method for the application of fire-control data of artillery.

Keywords: Neural Network; Orthogonal Arrays; Ballistic; Artillery

使用類神經網路與直交表於火砲射控數據之應用

陳一葳、杜耿銘、黃卓礽
空軍航空技術學院飛機工程系

摘要

火砲射擊精準度受到可控與不可控因素的影響，且影響戰爭的效果。火砲射擊控制的參數是依據射表在不同環境下做調整以獲得較佳的射擊精度，傳統人工化製作的射表需要耗費大量的彈藥與時間，現代化的射表則需要依賴精確的數學彈道模型。本研究利用類神經網路理論與直交表設計建立火砲彈道模型來預估火砲射程，類神經網路不需要複雜的數學模型便能建立火砲彈道模型，直交表的使用則可以減少對射擊數據的需求。研究所提的方法與傳統的迴歸分析及未使用直交表設計的類神經網路做比較，結果顯示提出的方法有較佳的預估精度且平均絕對誤差百分比可達1.422%。本研究著重在類神經網路與直交表的使用，以建立複雜的火砲彈道模型，並提供一個可行的方法應用在火砲射擊控制上。

關鍵字: 類神經網路、直交表、彈道、火砲

Nomenclature

v_j	Bias
Δv_j	Bias correction
T_k	Target output of neural network
F_e	Error function
F_{RMSE}	Cost function
i	Number of input layer neuron
j	Number of hidden layer neuron
k	Number of output layer neuron
N	Total number of neurons
Q	Total number of samples
w_{ji}	Weight
Δw_{ji}	Weight correction
x_i	Input value of input layer neuron
y_k	Output value from output layer neuron
y_j	Output value from hidden layer neuron
ε	Learning rate

1. INTRODUCTION

"Precision" and "accuracy" are the major factors to evaluate the artillery firepower and they are also the key points to influence war effectiveness. The army changes the projectile trajectory falling point of artillery according to the range tables. For external ballistic problems, the factors to affect the projectile trajectory are very complicated. For the controllable factors, such as the projectile weight, ammunition quantity, cannonball shape and so on, are set in the fabrication process. The projectile after leaving the muzzle will be affected by the uncontrollable factors such as initial velocity, air temperature, air pressure, relative humidity, wind velocity, wind direction, coriolis force and so on. These complicated factors increase the difficulty to build the artillery ballistic mathematical model. Traditionally, the man-made fabrication method to the range tables can obtain more precise information to set the fire-control data of artillery, but it costs much time and money and can't contain various environment conditions for firing. Therefore, the operation of man-made fabrication method is finite. The fabrication of modern range tables depends on the precise projectile motion equations and aerodynamic property. Even though many ballistic codes are utilized to make the artillery range tables now, the ballistic codes still need the precise ballistic parameters to obtain the precise range tables. For many applications, buying the foreign artillery and using the ammunition made by themselves is the tendency but the ammunition is not suitable for the range tables of original equipment manufacturer in firing. Accordingly, the major technique for improving the firing precision is to build a suitable artillery ballistic model for the application of fire-control data of artillery.

In some papers, the firing data or the range tables were utilized to build the artillery ballistic prediction equation by using the regression analysis method [1-3]. The method needs a lot of data to obtain the precise artillery ballistic model. Some study used the control theory and the equation of mass center motion to compute the resistance coefficient of ballistic with random wind [4]. The kalman filtering theory combined with the mass center ballistic model was also utilized to build the ballistic model in estimating the trajectory falling point [5]. The disadvantage of these proposed methods need a precise ballistic mathematical equation to obtain better prediction precision.

It is very difficult to build the mathematical model to describe the complicated relation of engineering system between input and output. The neural network is developed gradually and used extensively especially for the application on prediction [6, 7]. The quantity of learning sample will impact the training efficiency of neural network. For this reason, some studies combine design of experiments with neural network to reduce the quantity of learning sample to build effectively network model for fitting the true system. Wang et al. suggested that the orthogonal array experimental results are used to train the neural network, and the network model can be used to make prediction, interpolation, extrapolation and optimization [8]. The orthogonal array was also combined with the neural network and Taguchi-genetic algorithm to search for the global optimum of fabrication conditions. The proposed approach provided an effective and economical solution for process optimization [9]. Yingpin et al. also utilized the orthogonal array experimental results to be the training sample and constructed the back-propagation neural network (BPN) to simulate the feasible domain for seeking the optimal filter design. The proposed approach not only reduced the experimental runs but also obtained better results than the common design structure [10]. The neural network was often combined with Taguchi-design of experiment for the parameter optimization, experiment validation, analysis of variance and so on, and the orthogonal array was utilized to reduce the experimental runs [11]. The neural network does not need to build the complicated mathematical model and the experimental design can achieve the same result as the full factorial design even though it reduces the experimental runs. In this study, the neural network and orthogonal array are used to build an intelligent artillery ballistic model. The proposed orthogonal array neural network ballistic model is compared with the model built by the regression analysis method and non-orthogonal array neural network. The results show that the proposed method has better precision of range prediction.

2. FUNDAMENTAL THEORIES

2.1 Ballistics and Range Tables

Ballistics is the science that deals with the motion of projectiles. Modern scholars divide the subject into interior, exterior and terminal ballistics, which describe respectively, the propulsion, atmosphere flight, and target impact action of projectiles. The modern science of exterior ballistics has evolved as specialized branch of the dynamics of rigid body, moving under the influence of gravitational and aerodynamic forces. The building of mathematical motion equations of projectiles is to compute the trajectory of flight. The precision and feasibility of projectile motion equations mentioned above are dependent on the precise aerodynamic property and computation. Even if the most normal motion equation of point-mass trajectory, the cannonball shape, projectile weight, muzzle velocity, air density, wind velocity, air resistance, gravity in the atmosphere and so on, are also under consideration [12].

The fire-control information of artillery is determined by use of various range tables and equipment. These tables contain the fire-control information under standard conditions and data correcting for nonstanadard conditions, and they include the tabular range tables, graphical range tables, and graphical site tables. The tabular range tables are the basic source of firing data and offer the fire-control information in a tabular format. The data listed are based on standard conditions. The fabrication of traditional man-made range tables cost huge amount of ammunition and time, and it can’t contain the whole environment conditions for firing. The modern range tables are made by the development of ballistics, aerodynamics, mathematics and outstanding computer science, and that can reduce the consumption of ammunition and time. But, the precise modern range tables are based on the precise projectile motion equations and aerodynamic property.

In this paper, the input information (e. g. muzzle velocity, angle of departure and air temperature) and output information (firing data) are utilized to build an intelligent artillery ballistic model quickly by use of BPN. For the proposed method, the complicated projectile motion equations are unnecessary and it can be applied to generate the firing table.

2.2 Orthogonal Arrays

Orthogonal arrays were originally developed by Professor C. R. Rao in 1947 [13]. Scheme of orthogonal arrays means the experiment design is geometrically balanced and statistically independent. For any process, each level of factors has the same number of occurrences in the experimental matrix. The property of orthogonal arrays is to obtain the same effective information as the full factorial experiment, and the experimental runs will be reduced. It not only decreases the cost of experiment and time but obtains the statistical effect in less experimental runs. The type of orthogonal arrays include two-level, three-level and mixed-level, and they are usually shown in $L_A(B^C)$. A, B and C represent individually the number of experimental runs, level and factor.

In this paper the neural network is used to build the artillery ballistic model to predict precisely the range. Angle of departure, muzzle velocity, air temperature, air pressure, wind velocity, wind direction and relative humidity are the main variables to influence the range of the artillery. The wind includes following wind and cross wind. The wind direction includes downwind direction and upwind direction. The downwind direction means the same direction with the motion trajectory of projectile and the upwind direction is on the contrary. Because the relation between the artillery range (output) and the variables mentioned above (input) are very complicated, the three-level design are used to each variable except that the wind direction is in two-level design. $L_{36}(2^2\times3^7)$ is the mixed-level orthogonal array adopted in this paper.

2.3 Back-Propagation Neural Network

The artificial neural network attempts to imitate the nervous system of organism like human brain. It is composed of large amounts of various neurons. The computation can be carried out in a parallel and spread way to deal with large amount of data like the human nerve structure. It is difficult to explain the causation of engineering system in a mathematical model especially for the nonlinear system. The correlation model between input and output can be built by the neural network through the observation data and the complicated mathematical equation is unneeded. The performance of neural network would be improved if the training data are huge, widespread and great diversity. It is often utilized to substitute for the traditional regression analysis method to treat the high dimensional nonlinear problems [14].

BPN is the most popular neural network algorithm and the scheme is shown in Figure 1. BPN consists of input, hidden and output layers. The training process of neural network is composed of forward pass, error computation and error back-propagation. In the forward pass, a neuron driven by the input signal produces the output that differs from the actual or desired target output and leads to the error. Compute the error function and the gradient steepest descent method is applied to minimize the error function. The weight and bias will be modified by error back-propagation if the value of cost function doesn’t satisfy the goal error. The error signals are then back propagated through the network from output layer to input layer as a sequence of corrective adjustment called weight modification. The hidden layers, neurons, learning rate and transfer function influence the neural network deeply such as the training speed and convergence condition. The amount of hidden layers, neurons, learning rate and transfer function are modified properly in the training process to minimize the error. Figure 2 is the flow chart of BPN algorithm and the relative mathematical equations are described as follows:

$$y_j^n = f\left(\Phi_j^n\right) \tag{1}$$

$$\Phi_j^n = \sum_i w_{ji}^n y_i^{n-1} - v_j^n \tag{2}$$

$$F_e = \frac{1}{2}\sum_k \left(T_k - y_k\right)^2 \tag{3}$$

$$\Delta w_{ji} = -\varepsilon \frac{\partial F_e}{\partial w_{ji}} \tag{4}$$

$$w_{ji} = w_{ji} + \Delta w_{ji} \tag{5}$$

$$\Delta v_j = -\varepsilon \frac{\partial F_e}{\partial v_j} \tag{6}$$

$$v_j = v_j + \Delta v_j \tag{7}$$

3. FIRING DATA AND NEURAL NETWORK BALLISTIC MODEL

In this paper, the artillery firing data come from the range table of original equipment manufacturer which was built by a large number of firing tests. It is the middle-caliber artillery and the warhead's diameter is 40 mm. The range table has two types: one is anti-aircraft and the other is ground used in this study. Table 1 shows the firing data scope for building the neural network ballistic model and choose 36 firing data for training the network are arranged in $L_{36}(2^2\times3^7)$ orthogonal array (Table 2). All of the firing data used in this paper are needed to be normalized for training the neural network and the normalization equation is as follows:

$$I_2 = \frac{I_1 - I_{\mathrm{m}}}{I_M - I_{\mathrm{m}}} \times \left(D_M - D_{\mathrm{m}}\right) + D_{\mathrm{m}} \tag{8}$$

I_1, I_2: the original data and normalized data separately; I_m, I_M : the original data of minimum and maximum separately; D_m, D_M : the minimum and maximum separately in [0,1] domain.

The neural network ballistic model is built by the following steps:

Step 1: The variables (input) and the artillery range (output) according to $L_{36}(2^2\times3^7)$ orthogonal array are both normalized to train the BPN.

Step 2: Select the parameters of neural network as follows: hidden layers: 2; hidden layer neurons: (7, 4); learning rate: 0.1; transfer function: sigmoid function. Root mean square error (RMSE) is used as the cost function and the goal error is 0.004.

$$E_{RMSE} = \sqrt{\frac{\sum_{q=1}^{Q}\sum_{k=1}^{N}\left(d_k\left(q\right) - y_k\left(q\right)\right)^2}{QN}} \tag{9}$$

Step 3: Choose 36 firing data randomly to confirm the objectivity and usefulness of trained BPN besides the original 36 training firing data.

Step 4: The goal error will be modified properly according to the result to approach the purpose of error minimum and that the BPN range prediction will be close to the true range. As mentioned above, the neural network ballistic model is completed. Table 3 is the 36 firing data chosen randomly for verification.

4. RESULTS AND DISCUSSION

The performance of BP neural network combined with the orthogonal array is evaluated by 36 firing data chosen randomly. Besides, the regression analysis method and the BP neural network without using the orthogonal array will be utilized to build the artillery ballistic model for range prediction and compared with the proposed model. The regression analysis method utilizes the firing data in orthogonal array (Table 2) to build the artillery ballistic regression model (Model A). Figure 3 shows the normal probability distribution built by the residual of regression model. The residual distribution tends to a straight without outliers. The result proves the error distribution is normal and the regression model is suitable. Additionally, we choose another 36 training data (Table 4) which are widespread and great diversity to build the BP neural network without using the orthogonal array (Model B).

The mean absolute percentage error (M_e) between the prediction and the true is the index to evaluate the neural network ballistic model. The closer it gets to true ballistic model, the smaller M_e is.

$$M_e = \sum_{q=1}^{Q} \left| \frac{d(q) - y(q)}{d(q)} \right| \times 100\% \Big/ Q \tag{10}$$

The error comparison of range prediction for three different methods mentioned above is shown in Table 5. The proposed method has the most precise range prediction, and all of errors are less than 3% except no. 7, 10, 11 and 30. The M_e can approach to 1.422%. The next is Model B, M_e= 3.266%. The range prediction of Model A is the worst, M_e= 6.363%. Figure 4 is the range prediction error of three different methods. The simulation results prove the proposed method has a satisfied prediction performance of artillery range.

5. CONCLUSION

One of main factors to decide the war result is the firepower of artillery. The promotion of firing precision and accuracy for the artillery is always the research goal of national defense industry. The fabrication of range tables of man-made method cost huge amount of time and ammunition. The modern range tables need the precise projectile motion equations and aerodynamic property. In this paper, the neural network and orthogonal array are utilized to build an intelligent artillery ballistic model for range prediction in less firing data without the complicated mathematical equations. The simulation results proves the proposed method has the better prediction precision of artillery range than the traditional regression analysis method and the non-orthogonal array BPN, and the mean absolute percentage error can approach to 1.422%. The research result can offer a usable method to build the artillery ballistic model quickly for the application of fire-control data of artillery and reduce the development cost of national defense industry.

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Table 1. Firing data scope for building the neural network ballistic model

Variables	Minimum	Maximum
Angle of departure (^{o})	1	30
Muzzle velocity (m/s)	1000	1010
Air temperature (oC)	5	39
Relative humidity (%)	50%	100%
Air pressure (mb)	1002	1019
Wind velocity (m/s)	1.7	6.3
Wind direction	Downwind	Upwind

Note:
1. The standard muzzle velocity is 1005(m/s) and the data range is defined by the fabrication deviation of ammunition.
2. The range of atmosphere data is defined by Taiwan climate condition.

Table 2. 36 firing data arranged in $L_{36}(2^2\times3^7)$ orthogonal array

No.	Variables									Range (m)
	Following Wind Direction	Cross Wind Direction	Angle of Departure (degree)	Muzzle Velocity (m/s)	Relative Humidity (%)	Air Pressure (mba)	Air Temperature (^oC)	Following Wind Velocity (m/s)	Cross Wind Velocity (m/s)	
1	Upwind	Upwind	1	1000	50	1002	5	1.7	1.7	2498
2	Upwind	Upwind	15	1005	75	1010	22	4	4	9334
3	Upwind	Upwind	30	1010	100	1019	39	6.3	6.3	12108
4	Upwind	Upwind	1	1000	50	1002	22	4	4	2528
5	Upwind	Upwind	15	1005	75	1010	39	6.3	6.3	9677
6	Upwind	Upwind	30	1010	100	1019	5	1.7	1.7	11246
7	Upwind	Upwind	1	1000	75	1019	5	4	6.3	2483
8	Upwind	Upwind	15	1005	100	1002	22	6.3	1.7	9341
9	Upwind	Upwind	30	1010	50	1010	39	1.7	4	12272
10	Upwind	Downwind	1	1000	100	1010	5	6.3	4	2488
11	Upwind	Downwind	15	1005	50	1019	22	1.7	6.3	9322
12	Upwind	Downwind	30	1010	75	1002	39	4	1.7	12298
13	Upwind	Downwind	1	1005	100	1002	39	4	1.7	2592
14	Upwind	Downwind	15	1010	50	1010	5	6.3	4	8959
15	Upwind	Downwind	30	1000	75	1019	22	1.7	6.3	11630
16	Upwind	Downwind	1	1005	100	1010	5	1.7	6.3	2513
17	Upwind	Downwind	15	1010	50	1019	22	4	1.7	9303
18	Upwind	Downwind	30	1000	75	1002	39	6.3	4	12119
19	Downwind	Upwind	1	1005	50	1019	39	6.3	1.7	2583
20	Downwind	Upwind	15	1010	75	1002	5	1.7	4	9206
21	Downwind	Upwind	30	1000	100	1010	22	4	6.3	11963
22	Downwind	Upwind	1	1005	75	1019	39	1.7	4	2582
23	Downwind	Upwind	15	1010	100	1002	5	4	6.3	9267
24	Downwind	Upwind	30	1000	50	1010	22	6.3	1.7	12022
25	Downwind	Upwind	1	1010	75	1002	22	6.3	6.3	2582
26	Downwind	Upwind	15	1000	100	1010	39	1.7	1.7	9874
27	Downwind	Upwind	30	1005	50	1019	5	4	4	11432
28	Downwind	Downwind	1	1010	75	1010	22	1.7	1.7	2570
29	Downwind	Downwind	15	1000	100	1019	39	4	4	9875
30	Downwind	Downwind	30	1005	50	1002	5	6.3	6.3	11672
31	Downwind	Downwind	1	1010	100	1019	22	6.3	4	2570
32	Downwind	Downwind	15	1000	50	1002	39	1.7	6.3	9844
33	Downwind	Downwind	30	1005	75	1010	5	4	1.7	11513
34	Downwind	Downwind	1	1010	50	1010	39	4	6.3	2608
35	Downwind	Downwind	15	1000	75	1019	5	6.3	1.7	9141
36	Downwind	Downwind	30	1005	100	1002	22	1.7	4	11977

Table 3. 36 firing data chosen randomly for verification

No.	Variables									Range (m)
	Following Wind Direction	Cross Wind Direction	Angle of Departure (degree)	Muzzle Velocity (m/s)	Relative Humidity (%)	Air Pressure (mba)	Air Temperature (^{o}C)	Following Wind Velocity (m/s)	Cross Wind Velocity (m/s)	
1	Upwind	Upwind	2	1001	50	1002	36	3.7	6	3997
2	Upwind	Downwind	25	1003	60	1019	6	5	4	10557
3	Downwind	Downwind	15	1002	90	1003	18	1	3	9397
4	Upwind	Downwind	10	1010	55	1009	8	1	1	7848
5	Upwind	Upwind	21	1005	70	1018	5	4	5	10040
6	Upwind	Downwind	6	1000	60	1005	19	5	2	6450
7	Downwind	Downwind	5	1004	70	1010	25	3	3	6126
8	Downwind	Upwind	11	1009	82	1011	22	1	4	8414
9	Downwind	Downwind	27	1001	50	1009	6	1	2	11079
10	Upwind	Upwind	4	1009	80	1018	26	1	3.5	5554
11	Upwind	Downwind	3	1010	100	1004	24	2	3	4914
12	Upwind	Upwind	13	1000	74	1005	5	2	4	8642
13	Downwind	Downwind	26	1003	80	1007	10	1	3	11103
14	Downwind	Upwind	29	1005	95	1017	25	4	1	11931
15	Upwind	Downwind	30	1004	70	1016	32	3	1	11930
16	Downwind	Downwind	7	1006	90	1002	7	5	5	6887
17	Upwind	Upwind	17	1007	80	1008	15	1	3	9723
18	Downwind	Downwind	14	1010	75	1010	5	4.7	4	8996
19	Downwind	Downwind	20	1000	65	1005	20	2	3	10467
20	Upwind	Upwind	8	1001	60	1018	22	4	5	7245
21	Downwind	Upwind	19	1002	80	1019	30	4	6	10521
22	Downwind	Downwind	16	1009	90	1002	36	5	1	10224
23	Upwind	Upwind	9	1010	50	1006	37	3	2	7966
24	Upwind	Downwind	12	1000	85	1002	16	3.7	6	8483
25	Downwind	Upwind	22	1001	95	1010	31	6	6	11240
26	Upwind	Downwind	23	1009	79	1019	17	6	2	10593
27	Upwind	Downwind	10	1005	50	1010	30	1	1	8165
28	Downwind	Downwind	15	1004	80	1002	39	5	3	10002
29	Downwind	Downwind	10	1009	60	1005	26	2	2.4	8208
30	Downwind	Upwind	5	1010	80	1009	28	3	3	6206
31	Downwind	Upwind	18	1000	70	1006	31	5.5	2.4	10445
32	Downwind	Downwind	20	1005	90	1019	36	3.4	3.4	10901
33	Upwind	Downwind	28	1003	75	1016	28	2.5	4.2	11626
34	Upwind	Downwind	30	1008	95	1015	17	3.4	6	11527
35	Upwind	Upwind	21	1001	88	1006	26	6	1.3	10561
36	Downwind	Upwind	22	1000	86	1019	38	5.8	2	11360

Table 4. 36 firing data chosen additionally for comparison

No.	Following Wind Direction	Cross Wind Direction	Angle of Departure (°)	Muzzle Velocity (m/s)	Relative Humidity (%)	Air Pressure (mba)	Air Temperature (°C)	Following Wind Velocity (m/s)	Cross Wind Velocity (m/s)	Range (m)
1	Upwind	Upwind	28	1002	63	1013	9	3.5	5.1	11067
2	Upwind	Downwind	17	1001	77	1019	22	3.2	4.1	9705
3	Downwind	Upwind	21	1003	51	1018	18	4.2	3.3	10589
4	Upwind	Upwind	6	1000	50	1003	22	4	2.4	6501
5	Upwind	Upwind	29	1005	65	1014	19	4.9	6.1	11397
6	Downwind	Downwind	2	1004	89	1017	5	1.9	3.5	3860
7	Upwind	Downwind	8	1001	72	1018	7	4	3.3	7036
8	Downwind	Upwind	28	1005	100	1009	22	3.6	1.7	11786
9	Downwind	Upwind	30	1010	50	1010	39	1.7	5.1	12414
10	Upwind	Downwind	1	1002	53	1012	20	6	4	2523
11	Upwind	Downwind	19	1005	74	1019	22	4.3	2.1	10092
12	Upwind	Upwind	18	1010	80	1002	21	4.1	3.7	10039
13	Downwind	Downwind	9	1008	93	1003	39	3.9	2.2	8160
14	Downwind	Downwind	22	1004	85	1010	5	5.3	4.1	10052
15	Upwind	Downwind	1	1006	75	1016	22	1.7	6.1	2546
16	Upwind	Upwind	13	1005	74	1015	5	1.9	2.4	8538
17	Downwind	Downwind	15	1010	96	1019	22	4	1.9	9524
18	Upwind	Downwind	24	1006	75	1010	10	4.1	3.6	10659
19	Downwind	Upwind	16	1005	69	1019	9	6.1	1.9	9480
20	Upwind	Downwind	5	1007	82	1006	22	1.7	4.1	6089
21	Downwind	Downwind	27	1002	99	1008	17	4	5.7	11523
22	Downwind	Upwind	19	1005	75	1017	34	2.2	3.6	10606
23	Downwind	Upwind	6	1006	81	1004	5	4.9	6.3	6441
24	Upwind	Upwind	3	1000	50	1011	11	6	2	4716
25	Upwind	Upwind	4	1007	75	1002	12	5.8	3.5	5430
26	Downwind	Upwind	12	1000	95	1011	39	5.1	2.5	9100
27	Downwind	Upwind	16	1005	63	1018	5	4.6	4.4	9364
28	Downwind	Downwind	21	1009	75	1013	22	1.7	3.6	10704
29	Downwind	Downwind	4	1000	83	1007	14	4	3.8	5459
30	Upwind	Upwind	3	1005	65	1002	23	5.2	5.1	4858
31	Downwind	Downwind	23	1007	81	1012	13	6.3	2.4	10948
32	Upwind	Downwind	7	1003	64	1002	15	2.3	5.7	6875
33	Downwind	Downwind	10	1008	69	1010	16	4	3.9	8047
34	Downwind	Upwind	14	1006	74	1005	31	4.5	5.6	9524
35	Downwind	Downwind	20	1004	71	1006	24	6.3	5.5	10727
36	Downwind	Downwind	23	1009	60	1014	25	1.8	3.2	11093

Table 5. Error comparison of range prediction

No.	True Range (m)	Proposed Method Prediction (m)	Proposed Method Error (%)	Model A Prediction (m)	Model A Error (%)	Model B Prediction (m)	Model B Error (%)
1	3997	4052	1.376	3474	13.092	5131	28.371
2	10557	10588	0.294	11207	6.161	10822	2.510
3	9397	9561	1.74	9433	0.380	9555	1.681
4	7848	7885	0.467	7245	7.689	7906	0.739
5	10040	10057	0.171	10605	5.626	10077	0.369
6	6450	6328	1.897	5532	14.238	6673	3.457
7	6126	5905	3.606	4972	18.832	6195	1.126
8	8414	8479	0.769	7888	6.248	8486	0.856
9	11079	10972	0.962	11599	4.698	10947	1.191
10	5554	5262	5.253	4308	22.437	5481	1.314
11	4914	4671	4.945	3873	21.177	5599	13.940
12	8642	8786	1.658	8626	0.182	8800	1.828
13	11103	11007	0.872	11548	4.007	10882	1.991
14	11931	11605	2.727	11779	1.272	11372	4.685
15	11930	11682	2.079	11851	0.662	11612	2.666
16	6887	6823	0.929	5849	15.067	7458	8.291
17	9723	9838	1.179	9893	1.748	9717	0.062
18	8996	9102	1.18	8859	1.520	9126	1.445
19	10467	10528	0.584	10890	4.044	10408	0.564
20	7245	7223	0.301	6467	10.737	7164	1.118
21	10521	10501	0.193	10639	1.118	10336	1.758
22	10224	10180	0.426	9971	2.479	9986	2.328
23	7966	7840	1.58	7223	9.327	7707	3.251
24	8483	8624	1.656	8391	1.082	8900	4.916
25	11240	11098	1.258	11477	2.112	10697	4.831
26	10593	10617	0.236	11060	4.412	10695	0.963
27	8165	8171	0.063	7515	7.962	7991	2.131
28	10002	9958	0.445	9697	3.049	9825	1.770
29	8208	8155	0.649	7506	8.549	8190	0.219
30	6206	5917	4.652	5060	18.460	6436	3.706
31	10445	10503	0.553	10579	1.279	10235	2.011
32	10901	10743	1.448	10885	0.144	10523	3.468
33	11626	11439	1.606	11820	1.668	11403	1.918
34	11527	11379	1.282	11719	1.669	11407	1.041
35	10561	10538	0.218	11107	5.168	10448	1.070
36	11360	11140	1.941	11447	0.767	10913	3.935
MAPE (%) / SD		1.422/1.328		6.363/6.405		3.266/5.008	

Note: SD means standard deviation

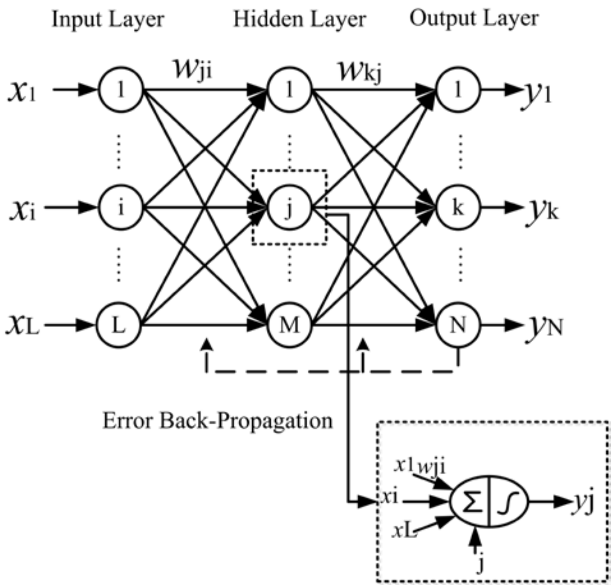

Fig. 1. Back-propagation neural network

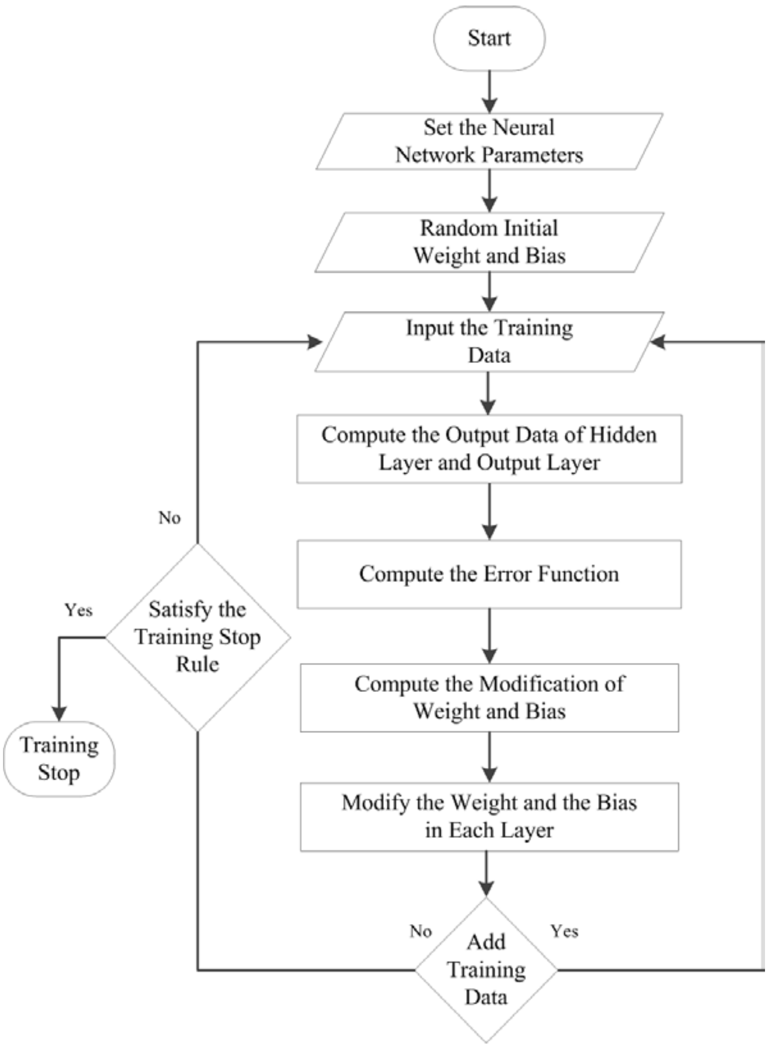

Fig. 2. Flow chart of BPN algorithm

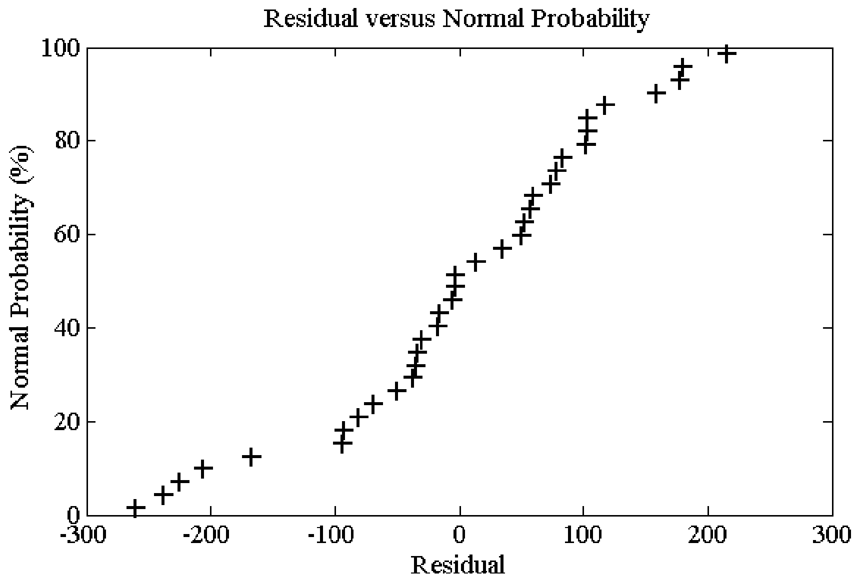

Fig. 3. Normal probability distribution

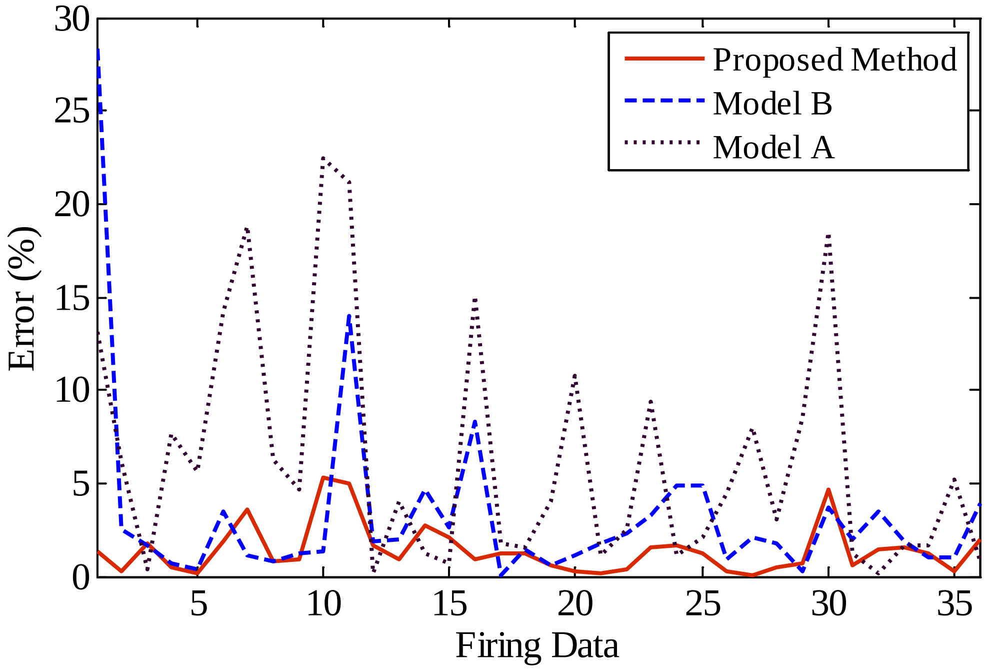

Fig. 4 Range prediction error of three different methods