MHD-Natural Convection over a Variable Wall Temperature Wedge

流經一可變壁溫度楔型面磁性自然對流熱傳遞

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Abstract

Boundary layer solutions for the case of variable wall temperature (VWT) are presented to investigate the steady, laminar, free convection flow of an electrically-conducting fluid over a wedge in the presence of a uniform applied magnetic field. The transformed nonlinear system of equations is solved by Keller box method. The range of values for the magnetic parameter ξ, the wedge angle parameter m, the Prandtl number Pr, and the exponent of VWT n are considered. The comparison of the dimensionless wall velocity and temperature gradients, i.e., $f''(\xi,0)$ and $-\theta'(\xi,0)$ for the case of uniform wall temperature (UWT) case, between our results and Watanabe and Pop's data is found to be very excellent.

Keywords：MHD-Natural Convection, Variable wall temperature wedge

摘要

在本文中吾人使用層流邊界層分析以探討流經一可變壁溫度楔型面磁性自然對流熱傳遞影響。偏微分方程式經轉換成非相似邊界層方程式後，再以凱勒盒子法解之。數值計算結果主要顯示：磁性參數ξ、契形角參數m、普蘭特數Pr、可變壁溫度指數n，對表面磨擦係數與Nusselt數之影響。

關鍵字：磁性自然對流、可變壁溫度楔型面。

1. Introduction

The problem of magnetohydrodynamic (MHD) flow and heat transfer process has many industrial applications such as MHD power generator, the cooling of nuclear reactors, and the heat exchanger design.

In the aspect of MHD-forced convection, Watanabe and Pop [1] investigated the heat transfer of an electrically conducting fluid over a semi-infinite isothermal flat plate in the presence of a transverse magnetic field by means of the difference-differential method including the viscous dissipation and stress work effects. Yih [2] extended the work of Watanabe and Pop [1] to study the heat transfer characteristics in MHD-forced convection flow adjacent to a non-isothermal wedge. In the aspect of MHD-free convection, the case of a semi-infinite isothermal vertical flat plate in the presence of uniform transverse magnetic field was investigated by Gupta [3] (series method), Sparrow and Cess [4] (series method), Lykoudis [5] (series and numerical methods), Riley [6] (integral method), and Kuiken [7] (singular perturbation). On the other hand, Wilks and Hunt [8] used the series and numerical solutions to present the case of uniform heat flux (UHF). Takhar and Soundalgekar [9] studied the dissipation effects on MHD-free convection flow past a semi-infinite isothermal vertical plate by the series method. The case of linear wall temperature (LWT) was numerically presented by Hossain [10]. This work is the extension of Takhar and Soundalgekar [9]. Gupta [11] investigated the magnetohydrodynamic free convection from a horizontal plate. Gray [12] presented the laminar plume above a line heat source in the presence of the transverse magnetic field. Similarity was shown to occur when the magnetic field strength varies as the $-2/5$ power of vertical distance from the source. Watanabe and Pop [13] considered the steady natural convection boundary flow of an electrically conducting fluid over an isothermal wedge by the difference-differential method. The problem of MHD-free convection over a cone or a wedge with variable wall temperature (VWT) was studied by Vajravelu and Nayfeh [14] using the local similarity solution. Yih [15] investigated the effect of uniform blowing/suction on MHD-natural convection over a horizontal cylinder maintained at a uniform wall temperature or uniform heat flux.

The aim of the present paper is to extend the work of Watanabe and Pop [13] to investigate the flow and heat transfer characteristics of MHD-free convection flow adjacent to a wedge with variable wall temperature (VWT).

2. Analysis

Consider the problem of MHD-free convection flow of an electrically conducting fluid over a wedge with the total angle Ω and the variable wall temperature (VWT) $[T_w(x) > T_\infty]$. The flow over the wedge is assumed to be two-dimensional, laminar, steady and incompressible. Neglecting the viscous dissipation in the energy equation due to the low velocity. The Hall effect is also neglected. Fluid properties are assumed to be constant except the density variation in the buoyancy force term. Introducing the boundary layer and Boussinesq approximations, the governing equations and the boundary conditions can be written as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T-T_\infty)\cos\left(\frac{\Omega}{2}\right)+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_o^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

$$y=0: u=0, \qquad v=0, \qquad T=T_w(x)=T_\infty+ax^n, \tag{4}$$

$$y\to\infty: u=0, \qquad T=T_\infty, \tag{5}$$

where x and y are coordinates measured along and normal to the wedge surface, respectively. u and v denote the velocity components in the x- and y- directions, respectively. The gravitational acceleration g is acting downward. β is the coefficient of thermal expansion. T is the temperature. ν is the kinematic viscosity. σ is the electrical conductivity. B_o is the externally imposed magnetic field in the y-direction. ρ is the density. α is the thermal diffusivity. a is the prescribed constant. n is the exponent of VWT.

The stream function ψ is defined by

$u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, therefore, the continuity equation is automatically satisfied.

Invoking the following dimensionless variables:

$$\xi = \frac{\sigma B_o^2 x^2}{\rho\nu}\sqrt{\frac{4}{Gr_x}} \quad (6.1)$$

$$\eta = \frac{y}{x}\sqrt[4]{\frac{Gr_x}{4}}\sqrt{\frac{m+1}{2}} \quad (6.2)$$

$$f(\xi,\eta) = \frac{\psi}{4\nu}\sqrt[4]{\frac{4}{Gr_x}}\sqrt{\frac{m+1}{2}} \quad (6.3)$$

$$\theta(\xi,\eta) = \frac{T - T_\infty}{T_w(x) - T_\infty} \quad (6.4)$$

$$Gr_x = \frac{g\beta[T_w(x) - T_\infty]x^3}{\nu^2} \quad (6.5)$$

and substituting Eq. (6) into Eqs. (1)-(5), we obtain

$$f''' + \frac{2(n+3)}{m+1}ff'' - \frac{4(n+1)}{m+1}(f')^2 + \frac{2}{m+1}\cos\left(\frac{\Omega}{2}\right)\theta - \frac{2}{m+1}\xi f' = \frac{4(1-n)}{m+1}\xi\left(f'\frac{\partial f'}{\partial\xi} - f''\frac{\partial f}{\partial\xi}\right), \quad (7)$$

$$\frac{1}{Pr}\theta'' + \frac{2(n+3)}{m+1}f\theta' - 8nf'\theta = \frac{4(1-n)}{m+1}\xi\left(f'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial f}{\partial\xi}\right). \quad (8)$$

The boundary conditions are defined as follows:

$$\eta = 0: f' = 0, \quad f = 0, \quad \theta = 1, \quad (9)$$

$$\eta_\infty \to \infty: f' = 0, \quad \theta = 0. \quad (10)$$

In addition, the velocity components are

$$u = \frac{4\nu}{x}\sqrt{\frac{Gr_x}{4}}f' = U_c f', \quad (11)$$

$$v = -\frac{\nu}{x}\sqrt{\frac{2}{m+1}}\sqrt[4]{\frac{Gr_x}{4}}\begin{bmatrix}(n+3)f + \\ 2(1-n)\xi\frac{\partial f}{\partial\xi} - (1-n)\eta f'\end{bmatrix}. \quad (12)$$

Here, $m = \Omega/(2\pi-\Omega)$ is the wedge angle parameter. ξ is the magnetic parameter. $Pr = \nu/\alpha$ is the Prandtl number. U_c is the reference convective velocity.

In the above equations, the primes denote differentiation with respect to η. The physical quantities of physical interest include the local skin friction coefficient and the local Nusselt number, which are given by

$$\frac{1}{2}C_f\sqrt[4]{\frac{Gr_x}{4}} = \frac{1}{4}\sqrt{\frac{m+1}{2}}f''(\xi,0) \quad \text{and}$$

$$Nu_x\sqrt[4]{\frac{4}{Gr_x}} = \sqrt{\frac{m+1}{2}}[-\theta'(\xi,0)]. \quad (13)$$

It may be noticed that for n = 0 (UWT), Eqs. (7)-(8) are reduced to those of Watanabe and Pop [13] where a nonsimilar solution was obtained previously. It is also observed that similar equations are obtained for the case of n = 1 (LWT) or $\xi = 0$.

3. Results and discussion

The present analysis integrates the system of Eqs. (7)-(8) by the implicit finite difference approximation together with the modified Keller box method of Cebeci and Bradshaw [16]. For the sake of brevity, the numerical method is not described. Computations were carried out on a personal computer with $\Delta\xi = 0.1$; the first step size $\Delta\eta_1 = 0.001$. The variable grid parameter is chosen 1.001 and the value of η_∞ is adjusted for Pr. The iterative procedure is stopped to give the final velocity and temperature distributions when the errors in computing the $|f''_w|$ and $|\theta'_w|$ in the next procedure become less than 10^{-5}.

Numerical results are presented for ξ ranging from 0 to 3, n ranging from 0 to 1, Pr ranging from 0.0001 to 1, and m ranging from 0.0909 to 0.3333.

In order to verify the accuracy of our

present method, we have compared our results with those of Watanabe and Pop [13]. Table 1 shows the comparison of the values of $f''(\xi,0)$ and $-\theta'(\xi,0)$ (UWT) for various values of ξ with m = 0.2, n = 0, Pr = 0.03. Table 2 lists the comparison of the values of $f''(\xi,0)$ and $-\theta'(\xi,0)$ (UWT) for various values of Pr with m = 0.2, n = 0, ξ = 1.0. Table 3 shows the comparison of the values of $f''(\xi,0)$ and $-\theta'(\xi,0)$ (UWT) for various values of m with n = 0, ξ = 1.0, Pr = 0.03. The comparison in the above cases is found to be in good agreement, as shown in Tables 1-3. Table 4 lists the values of $f''(\xi,0)$ and $-\theta'(\xi,0)$ (VWT) for various values of n with ξ = 3.0, m = 0.3333, Pr = 0.01. Increasing the exponent of VWT n decreases the dimensionless wall velocity gradient and increases the dimensionless wall temperature gradient. Thus, the local skin friction coefficient decreases and the local Nusselt number enhances with increasing the exponent of VWT n.

4. Conclusions

A laminar boundary layer analysis is used to investigate the flow and heat transfer characteristics of MHD-natural convection over a wedge. The surface of the wedge is maintained at a variable wall temperature (VWT). The nonsimilar governing equations are obtained by using a suitable transformation and solved by Keller box method. Numerical results for the dimensionless velocity profiles, the dimensionless temperature profiles, the local skin friction coefficient, and the local Nusselt number are presented for various values of the magnetic parameter ξ, the exponent of VWT n, the Prandtl number Pr, and the wedge angle parameter m. Increasing the Prandtl number Pr, the exponent of VWT n, the wedge angle parameter m, and the magnetic parameter ξ decreases the local skin friction coefficient. The local Nusselt number increases (decreases) with an increase in n and Pr (ξ and m).

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Table 1 Comparison of the values of $f''(\xi,0)$ and $-\theta'(\xi,0)$ (UWT) for various values of ξ with $m = 0.2$, $n = 0$, $Pr = 0.03$

ξ	$f''(\xi,0)$		$-\theta'(\xi,0)$	
	Watanabe and Pop [13]	Present results	Watanabe and Pop [13]	Present results
0.0	1.08756	1.08758	0.16767	0.16771
0.5	0.96845	0.96852	0.15564	0.15564
1.0	0.86767	0.86774	0.14410	0.14410
1.5	0.78264	0.78361	0.13237	0.13325
2.0	0.71416	0.71404	0.12312	0.12325
2.5	0.65670	0.65675	0.11424	0.11421
3.0	0.60937	0.60947	0.10633	0.10626

Table 2 Comparison of the values of $f''(\xi,0)$ and $-\theta'(\xi,0)$ (UWT) for three values of Pr with $m = 0.2$, $n = 0$, $\xi = 1.0$

Pr	$f''(\xi,0)$		$-\theta'(\xi,0)$	
	Watanabe and Pop [13]	Present results	Watanabe and Pop [13]	Present results
0.02	0.88303	0.88288	0.11930	0.11934
0.04	0.85551	0.85558	0.16445	0.16444
0.05	0.84522	0.84529	0.18200	0.18198

Table 3 Comparison of the values of $f''(\xi,0)$ and $-\theta'(\xi,0)$ (UWT) for two values of m with

$n = 0$, $\xi = 1.0$, $Pr = 0.03$

m	$f''(\xi,0)$		$-\theta'(\xi,0)$	
	Watanabe and Pop [13]	Present results	Watanabe and Pop [13]	Present results
0.0909	0.99891	0.99899	0.15661	0.15660
0.3333	0.69132	0.69139	0.12781	0.12781

Table 4 The values of $f''(\xi,0)$ and $-\theta'(\xi,0)$ (VWT) for various values of n with

$\xi = 3.0$, $m = 0.3333$, $Pr = 0.01$

n	$f''(\xi,0)$	$-\theta'(\xi,0)$
0.0	0.48499	0.05419
0.25	0.47336	0.06980
0.5	0.46425	0.08209
0.75	0.45670	0.09225
1.0	0.45020	0.10094