Static Analysis of a Self-anchored Cable-stayed-suspension Bridge with Optimal Cable Tensions

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ABSTRACT

A self-anchored cable-stayed-suspension bridge is a new type of cable-supported bridge developed from the ideas of self-anchored suspension bridges and cable-stayed bridges. It has the advantages of cost reduction, reasonable mechanical behaviors, good aerostatic stability and strong foundation adaptability. In the current study, a review of previous studies on the development of self-anchored cable-stayed-suspension bridges is conducted, along with a corresponding theoretical analysis. Then, relying on experiences of designing main cable curves for suspension bridges and adjusting cable tensions in cable-stayed bridges, a two-stage approach is proposed to obtain the reasonable dead-load state, which derived from a nonlinear FEM and constraint relaxation quadratic programming method. Finally, through analysis of the Zhuanghe Jianshe Bridge, design problems that require special attention are pointed out and a reference basis for further application and design of self-anchored cable-stayed-suspension bridges is provided.

Keywords: self-anchored cable-stayed-suspension bridge, reasonable dead-load state, quadratic programming method, constraint relaxation

自錨式吊拉組合橋考量索力優化之靜態力學分析

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摘　　要

自錨式吊拉組合橋是在現代自錨式懸索橋和斜張橋基礎上形成的一種新型纜索承重橋梁，具有造價節省，受力合理，抗風性能好，地基適應性佳等優點。本文首先介紹這種橋型的發展狀況，研究其分析理論，然後綜合懸索橋主纜線形設計與斜張橋索力調整方面的經驗，基於非線性有限元素法和約束鬆弛的二次規劃法提出了獲取其合理靜載狀態的兩階段迴圈法，最後經莊河建設大橋的分析指出設計中應注意的問題，為自錨式吊拉組合橋的廣泛應用提供依據。

關鍵詞：自錨式吊拉組合橋，合理靜載狀態，二次規劃法，約束鬆弛

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Ⅰ. INTRODUCTION

A cable-stayed-suspension bridge is a combination of a cable-stayed bridge and a suspension bridge [1,2]. This kind of bridge offers good mechanical behaviors and prominent economic benefits. Moreover, the span, rigidity and stability can also be improved. In addition, the height of pylons and the volume of anchorages are reduced, and the safety of bridges is strengthened during the construction and operation stages. Some engineers are aware of the advantages of cable-stayed-suspension bridges as long-span bridges, and have frequently proposed using this type of bridge as sea-crossing bridges, e.g. the Messina Strait Bridge in Italy, the Great Belt Bridge in Denmark, the Izmit Bridge in Turkey, the Lingdingyang Bridge and the Runyang Bridge in China. Despite that, cable-stayed-suspension bridges have not been widely accepted for certain reasons, such as discontinuous configurations and complex mechanical behaviors. Instead, suspension bridges and cable-stayed bridges were used because of concise configuration and mature technologies of these traditional bridge designs. Consequently, cable-stayed-suspension bridges have not gained any further development during the last several decades.

Self-anchored cable-stayed-suspension bridges are a new type of cable-supported bridge, originating from modern self-anchored suspension bridges and cable-stayed bridges [3]. Compared with traditional earth-anchored cable-stayed-suspension bridges, self-anchored cable-stayed-suspension bridges have main cables anchored at the end of the stiffening girders, eliminating the difficulty and risk of massive end anchorage construction. The stayed and suspended parts of the stiffening girder act as compression-bending members, improving the continuity of girder mechanical behaviors. This kind of bridge can be flexibly arranged for multi-tower structures with less restriction of geological condition. The advantages of cable-stayed-suspension bridges, such as reduced construction risk and low cost, are also preserved [4]. The difference between a self-anchored cable-stayed-suspension bridge and an earth-anchored bridge is in the anchor location; that is, from earth to the end of the bridge deck. Moreover, the mechanical characteristics and construction methods also differ greatly [5]. The stayed and suspended parts of earth-anchored systems are non-intercoupling, and they can be analyzed as independent structures. However, this is not true of self-anchored systems. Additionally, with design methods totally different from cable-stayed bridges and suspension bridges, the first problem is how to coordinate the two design methods to obtain the reasonable dead-load state of the self-anchored cable-stayed-suspension bridge.

Ⅱ. ORIGINS AND DEVELOPMENT

At the end of 20th century, sea-crossing bridges were booming. Many countries located beside straits proposed and carried through successively the construction of sea-crossing bridges. Sea-crossing bridges are different from normal long-span bridges, because they must have enough span to satisfy long-term navigation, with foundations in deep water or weak soil areas and sites usually in strong typhoon regions. For suspension bridges, massive end anchorages pose great difficulties in construction and are very expensive, not to mention poor wind stability. For cable-stayed bridges, both span and pylon height are limited, and the risk of long cantilever construction is great. Therefore, when designing sea-crossing bridges, the major problem is how to increase spans, lower pylons, reduce costs and ensure safety.

With long-term theoretical research and engineering experience in self-anchored suspension bridges, researchers from the Bridge Institute in DUT (Dalian University of Technology) conceived a preliminary model for a self-anchored cable-stayed-suspension bridge during scheme design for a sea-crossing bridge. Thereafter, two bridges of this type were designed. One is the Zhuanghe Jianshe Bridge with concrete girders, and the other is the Jinzhou Bay Bridge with steel-concrete hybrid girders.

2.1 Zhuanghe Jianshe Bridge

Completed in 2008, the Zhuanghe Jianshe Bridge in Dalian, Liaoning Province, is the first concrete self-anchored cable-stayed-suspension bridge in China, see Fig. 1. This bridge adopted the modified Dischinger system. The entire bridge is 202 m long, and has a central span of 110 m and two side spans of 46 m. The suspended part and the stayed part of the central span are 38.4 m and 39 m, respectively; and the ratio of sag to suspended span is 1/5. In addition, cross hangers were employed at the suspended and stayed conjunction parts. The pylon and stiffening girders are both of concrete structures. The construction scheme is as follows. First, the pylons were constructed by the slip form method; then, the stiffening girder was constructed on the falseworks before the main cables were erected. Finally, the hangers and stay cables were tensioned under construction control to transfer deck loads from temporary supports to cables.

Fig. 1 Zhuanghe Jianshe Bridge.

The Zhuanghe Jianshe Bridge represents the transformation of a self-anchored cable-stayed-suspension bridge from conception to reality. Although the construction method followed that for self-anchored suspension bridges (i.e., the stiffening girder was erected before the main cables) the design, construction control and loading test provide data of mechanical characteristics of self-anchored cable-stayed-suspension bridges, laying a foundation for development to long span.

2.2 Jinzhou Bay Bridge

In 2006, in a bridge design competition for the Jinzhou Bay in Dalian, Liaoning Province, a self-anchored cable-stayed-suspension bridge was awarded the first prize due to its peculiar charm. The design was chosen for the Jinzhou Bay Bridge, without modification. The Jinzhou Bay Bridge is a double-pylon, double-cable-plane bridge, adopting the modified Dischinger system. Two groups of cross hangers are adopted at the conjunction part of the suspended and stayed parts. The bridge has a total length of 664 m, consisting of a central span of 400.0 m and two side spans of 132.0 m. The ratio of cable sag to suspended span part is 1/6.667. In the central span, the length of the suspended part adopting steel box girders is 136.0 m, and that of the stayed part adopting concrete box girders is 140.0 m. With the temporary earth anchorages, the suspended part can utilize the traditional construction methods of suspension bridges, i.e. main cables erected before stiffening girders. The cable-stayed decks are constructed as balanced cantilevers. After the steel girder consolidates with the concrete girders, the ends of the main cables are transferred from temporary earth anchorages to the deck, with the bridge changing from an earth-anchored system into a self-anchored system.

The Jinzhou Bay Bridge is an attempt at a large-span self-anchored cable-stayed-suspension bridge. A series of constructional methods are adopted to avoid the problems of hanger fatigue damage and the discontinuity of internal force and deformation. With the help of temporary earth anchorages, the girder erection of the suspended part gets rid of falseworks. Therefore, the construction technologies of traditional suspension bridges can be used. The Jinzhou Bay Bridge is a real large-span self-anchored cable-stayed-suspension bridge.

Ⅲ. THEORETICAL ANALYSIS

A self-anchored cable-stayed-suspension bridge is a complex cable-supported system. It cannot be analyzed directly by the theories for self-anchored suspension bridges [6,7]. We must consider the high-order statistically determinate character of cable-stayed bridges. The deflection theory is the most widely applied method that takes the effect of structural deformation into account, requiring few parameters and short programs in the stage of scheme and preliminary design. In order to analyze the effect of main cables in a self-anchored cable-stayed-suspension bridge, the following three assumptions are proposed for the deflection theory:

(1) The distribution of dead load is uniform throughout the bridge. The cable coordinates of the mid span for hangers are parabolic under dead load.

(2) The stiffness of pylons is neglected and the stay cables are simplified as elastic supports along the girder.

(3) The hangers remain vertical under load.

With these assumptions and the notations shown in Fig. 2, the bending moments of the suspended span under live load can be given as:

$$\begin{aligned} M = M_d + M_l - (H_d + H_l)(y + v_1) \\ - (H_d + H_l)(c - v_2) \end{aligned} \quad (1)$$

Where M_d, M_l are dead-load and live-load moments of the unsuspended girder respectively; H_d, H_l are the horizontal component of cable tension under dead load and live load respectively; y is the ordinate of the main span cable curve at the location of desired moment; v_1, v_2 are the vertical deflection of the main cable and the girder under live load, respectively; and c is the girder camber under dead load.

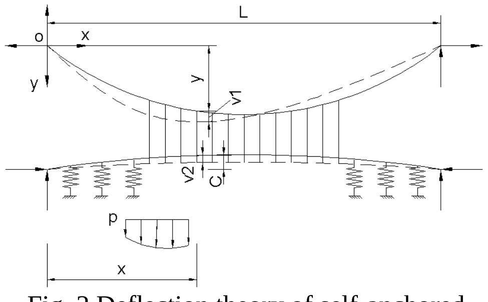

Fig. 2 Deflection theory of self-anchored cable-stayed-suspension bridges.

With Assumption 1, the equilibrium condition under dead load is

$$M_d = H_d(y + c) \quad (2)$$

Substituting Eq. 2 into Eq. 1 yields

$$M = M_l - H_l(y + c) - (H_l + H_d)(v_1 - v_2) \quad (3)$$

When the length of the stayed span turns to zero, v_1 and v_2 will become zero accordingly. and Eq.3 can be written as,

$$M = M_l - H_l(y + c) \quad (4)$$

Equation 4 expresses the deflection theory for a self-anchored suspension bridge [5]. Compared with Eq. 3, Eq. 4 does not have $-(H_l + H_d)(v_1 - v_2)$. This is caused by the end displacement of main cables without hangers and the elastic supports of the stay cables. Nonlinear effects always exist, regardless whether the span is long or short. Therefore, applying the elastic theory causes unpredictable errors when analyzing self-anchored cable-stayed-suspension bridges. However, the deflection theory, as shown in Eq.4, is cumbersome when applied to more complex models. Thus, using a nonlinear finite element model seems more reliable.

Ⅳ. REASONABLE DEAD-LOAD STATE

4.1 Main Cable Curve under Dead load

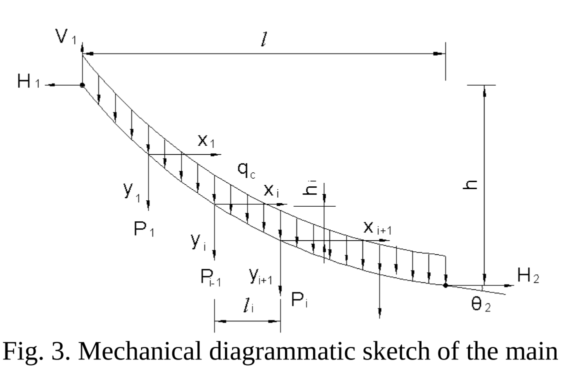

Fig. 3. Mechanical diagrammatic sketch of the main cable.

Main cables will bear two types of loads after a bridge is finished. One is the uniformly distributed self-weight of cables along the arc, including wire strands and wrappings. The other is the point load transferred from the i^{th} hanger, including the self-weight of stiffening girders, pavement and cable bands. The curve of the cable between the hangers is catenary. Therefore, the main cable is a series of catenaries secluded by the hangers [8,9], as shown in Fig 3.

For the i^{th} cable segment, the equilibrium equation is:

$$\frac{d^2 y_i}{dx_i^2} = -\frac{q_c}{H}\sqrt{1+\left(\frac{dy_i}{dx_i}\right)^2} \tag{5}$$

Substituting the boundary conditions into (5) yields

$$y_i = a1_i[ch(\frac{x}{a1_i} + a2_i)] + a3_i \tag{6}$$

Where $a1_i = -\frac{H}{q_c}$;

$$a2_i = sh^{-1}\frac{h_i}{2a1_i\, sh(\frac{l_i}{2a1_i})} - \frac{l_i}{2a1_i};$$

$$a3_i = -a1_i\, ch(a2_i)$$

Thus, the overall equilibrium equations of the main cable can be written as:

$$\begin{cases} H_1 = H_2 \\ V_1 = q_c \sum_{i=1}^{n} s_i + \sum_{i=1}^{n} P_i + H_2 \tan\theta_2 \end{cases} \tag{7}$$

The compatibility equation for the cable is:

$$\sum_{i=1}^{n} h_i = h \tag{8}$$

Where s_i is the cable length of the i^{th} segment, and

$$\begin{aligned} s_i &= \int_0^l ds = \int_0^l \sqrt{1+(y')^2}\,dx \\ &= a1_i[sh(\frac{l_i}{a1_i} + a2_i) - sh(a2_i)] \end{aligned} \tag{9}$$

At the position of concentrated loads, the equilibrium equation is

$$H\, y_i'\big|_{x_i = l_i} - H\, y_{i+1}'\big|_{x_i = 0} = P_i$$

or

$$H[sh(\frac{l_i}{a1_i} + a2_i) - sh(a2_{i+1})] = P_i \tag{10}$$

Where l_i, h_i are the horizontal and vertical distances between two adjacent concentrated loads; with l, h being the overall horizontal and vertical distances respectively. According to Eqs. 6, 7 and 10, the main cable curve under dead load can be estimated by an iterative method.

4.2 Constraint Relaxation Quadratic Programming Method

In the application of the constraint relaxation quadratic programming method for cable adjustment, the corresponding mechanical parameters should first be defined. Next, the design variables, constraint conditions and objective functions are determined [10]. In practice, the following major issues should be taken into account [11].

(1) Girder and pylons bending moments. These moments are the goal of cable tension optimization. Different weight values are chosen to balance the difference in carrying capacity between the girder section and pylon section. When both live-load and prestressing effect are considered in design, the center of moment envelope of the two effects will be included in the initial moments, before the dead-load bending strain energy function is defined.

(2) Cable tensions. Tensions of the stay cables may be nonuniform because of the cross hangers. To satisfy the uniformity requirement, upper and lower limits of cable tensions are set different for different parts. It should be noted that since the side span lacks hangers the main cables of the side span can also be regarded as special end anchors, having an important effect on the forces of pylons, girders and anchors, and constitute the stayed part during the process of cable tension adjustment.

(3) Support reactions. There should be enough pressure to avoid negative support reaction of the side piers and auxiliary piers under live load, and this is usually achieved by bearing overburden or tension-compression bearings.

The aforementioned upper and lower limits of constraint boundary are usually within a reasonable range as predefined by engineering experience or corresponding engineering specifications, and the problem is solved by the quadratic programming method. However, the quadratic programming model takes only one definite value [12]. For small values, the results of objective functions are approximately accurate, but the feasible region is reduced and convergence cannot be guaranteed. For large values, the results are too rough. In practice, the main constraint conditions must be strictly satisfied, while secondary constraint conditions if not satisfied are controlled by other methods. If none of the constraint conditions can be satisfied, the structural parameters should be redefined. Thus, different relaxation coefficients are introduced into the constraint conditions, which can simultaneously satisfy the requirements of precision and global convergence. This can be achieved by setting a small step size for main constraint coefficients and a large step size for secondary ones. The constraint relaxation quadratic programming method can be written as:

$$\begin{aligned} &\min.\ U = \int \rho(s)\frac{[M(s)-M_0(s)]^2}{2EI}ds \\ &s.t. \quad \alpha_1 \underline{M_t^i} \le M_t^i \le \alpha_2 \overline{M_t^i} \quad (i=1,2,\cdots,n1) \\ &\qquad \alpha_3 \underline{M_b^j} \le M_b^j \le \alpha_4 \overline{M_b^j} \quad (j=1,2,\cdots,n2) \\ &\qquad \alpha_5 \underline{R^k} \le R^k \le \alpha_6 \overline{R^k} \quad (k=1,2,\cdots,n3) \\ &\qquad \alpha_7 \underline{F^l} \le F^l \le \alpha_8 \overline{F^l} \quad (l=1,2,\cdots,n4) \end{aligned} \tag{11}$$

Where U is the bending strain energy of the girder and pylons, and denotes the objective function; $M(s)$ is the moment under cable tension F; $M_0(s)$ is the center of moment envelope for live-load and prestressing effects; E is the elasticity modulus; I is the flexural rigidity; $\rho(s)$ is the weight value of the girder and pylons. M_t^i is the moment of pylon reference point i under cable tension F; M_b^j is the moment of girder reference point j under cable tension F; R^l is the support reaction of reference point l under cable tension F; F^m is the m^{th} cable tension. Underlining denotes a lower limit, and overlining represents an upper limit. *n1*, *n2*, *n3* and *n4* are the total numbers of reference points of pylons, girder, support reactions and cables correspondingly. $\alpha_1,\alpha_2,\cdots,\alpha_8$ are the relaxation coefficients.

4.3 Two-stage Approach

The cables can be adjusted within a certain range, which could make the distribution of dead loads more reasonable. After this step is finished, the analysis that follows will usually adhere to a specific procedure that cannot be adjusted. Thus, the distribution of dead loads is an important factor for judging whether the design is good or not. In self-anchored cable-stayed-suspension bridges, not only can the stay cables be adjusted as in the case of cable-stayed bridges, but coordinates of the main cables and unstrained lengths can also be adjusted as in the case of suspension bridges. However, it is very difficult to adjust the cable tension, cable coordinates and unstrained lengths simultaneously. The two-stage approach separates the design of the main cables for suspension bridges from the adjustment of cable tensions in cable-stayed bridges. That is, the coordinates of both main cables and unstrained lengths are first computed, and the stay cables are then adjusted to obtain a reasonable dead-load state. Therefore, traditional methods can be readopted in each stage. The steps of the algorithm for the two-stage approach are listed below:

Step 1: Estimate initial structural dimensions and initial weight values of the girder and pylons. Assume that the relaxation coefficients are 1.0.

Step 2: Suppose the initial tensions of hangers is the self-weight of girder between hangers and cable bands. The coordinates of main cables and unstressed lengths are calculated by the curve design theory for main cables. The first stage is completed.

Step 3: Restrain temporarily the vertical degrees-of-freedom of main cables at the position of hangers, perform the minimum bending energy method or other methods to get the initial dead-load state. Calculate the center of moment envelop for live-load and prestressing effects to generate the mathematical model.

Step 4: Obtain the necessary and sufficient conditions of the quadratic programming problem by the Kuhn-Tucker condition and solve the problem with Lemke’s method.

Step 5: Stop and output results if there is an optimum solution. Otherwise, modify relaxation coefficients according to $\alpha_i = \alpha_i + \lambda_i$, where λ_i is the step size of relaxation coefficients, which are determined by the importance of constraint conditions. Go to Step 3 and repeat the procedure three or five times to complete the second stage.

Step 6: If the L2-norm of the temporary restraint reactions is less than the tolerance ξ, the reasonable dead-load state has been obtained. Otherwise, restart a new procedure with the adjusted hanger tensions as new initial values.

The analysis process is shown in Fig. 4.

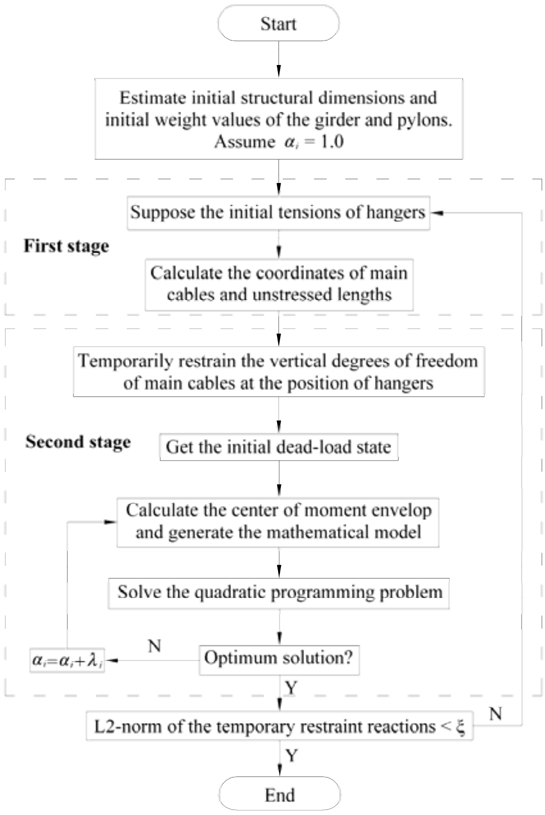

Fig.4. Flow chart for the two-stage approach method.

Ⅴ. NUMERICAL EXAMPLE

Figure 5 shows the major structural dimensions of the Zhuanghe Jianshe Bridge. The diameters of the main cables, stay cables and hangers are 71.8 mm, 37.1 mm and 45.3 mm, respectively. The spacing of the hangers and stay cables on the girder are both 6.4 m, and the spacing of the stay cables on the pylons is 3.2 m. The concrete deck consists of two longitudinal concrete edge girders. The width of the deck is 28.60 m. A central reservation with a total width of 4.50 m is set up in the center of the bridge. There are two traffic lanes with a total width of 8.50 m on each side of the median separator, with a footpath of 2 m wide and a cable zone of 1.55m wide on the outside of each lane. The height of the girders is 2.17 m, the thickness of the deck is 0.30 m, and the transverse diaphragms have a spacing of 6.4 m. C1-C12 and C20-C31 are stay cables, and C13-C19 are hangers.

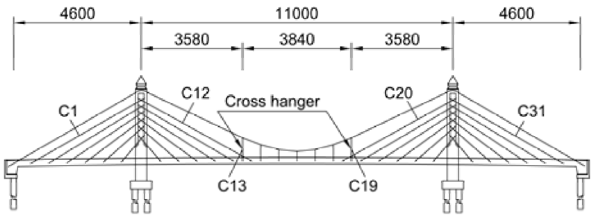

Fig. 5. Structural dimensions of the Zhuanghe Jianshe Bridge. (Units: cm)

In view of constraint conditions such as pylon bending moments, cable tensions and support reactions, the relaxation values of the constraint boundary are distinguished by actual engineering. The pylon bending moments set a larger relaxation value, and the step size of relaxation coefficients is 0.1. The cable tensions set a smaller value, and the step size is 0.05. The minimum value of the girder bending strain energy can be obtained when the constraint conditions are all satisfied. The coordinates of the main cables and unstressed lengths are then adjusted after cable tensions are optimized. The results reach stable values after three iterations. The reasonable dead-load state of the Zhuanghe Jianshe Bridge is calculated by the two-stage approach method. The comparisons between the reasonable dead-load state and the design values are shown in Figs. 6-9.

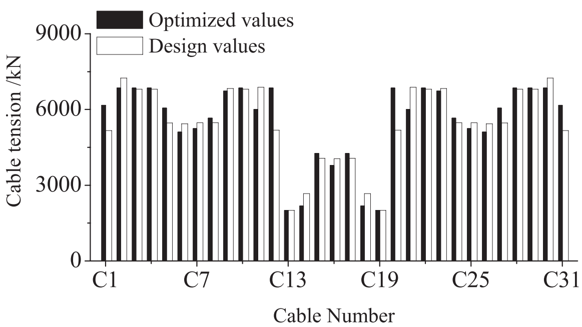

Fig.6. Comparison of cable tensions.

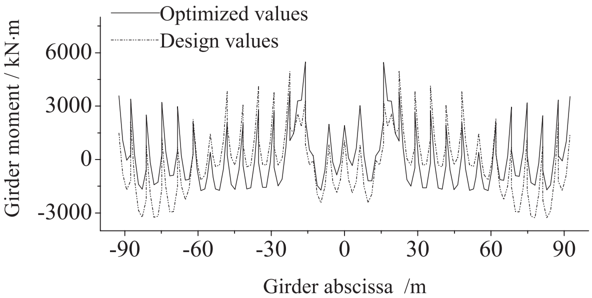

Fig.7. Comparison of girder moments.

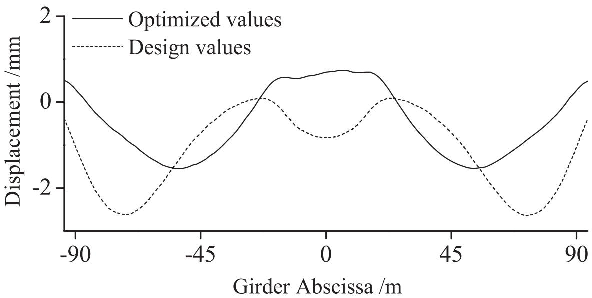

Fig.8. Comparison of girder displacements.

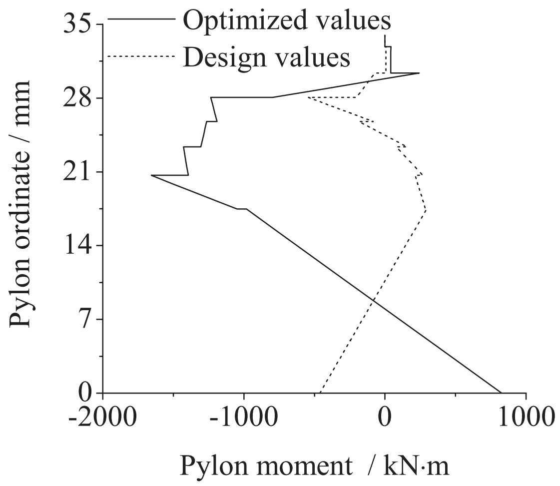

Fig.9. Comparison of pylon moments.

The distribution of internal force in the stay cables and hangers is shown in Fig. 6. Compared with the design values, the uniformity of optimized cable tensions is improved, indicating that the tensions of stay cables are greater than those of hangers, the cable tensions close to the pylon supports are smaller, and the tensions of cross hangers are half that of normal hangers. Figures 7 and 8 show that the peak value of girder moment at the side span (except end anchor parts of the girder) is almost the same as that at the mid span; the maximum positive moment of the design values is 3278 $\text{kN}\cdot\text{m}$, and the corresponding optimized values is 1743 $\text{kN}\cdot\text{m}$, which is 53% of the design values. The moment at the cross hangers is complex with or without optimization; and the girder displacement under dead load is small, leading to the simplified fabrication technology of the girder. Although the maximum moment of pylons is slightly enlarged, it is still at a low stress level, as seen in Fig. 9. This is because the optimization made full use of the flexural rigidity of pylons, with a large step size of relaxation coefficients adopted in the constraint relaxation quadratic programming method, and proper enlarged moments of the pylons help optimize other structural performances.

Ⅵ. CONCLUSIONS

From this study of self-anchored cable-stayed-suspension bridges, the characteristics of this kind of bridge are summarized as follows.

(1) Applying the elastics theory to analyze self-anchored cable-stayed-suspension bridges leads to unpredictable errors.
(2) The constraint relaxation quadratic programming method is more practical, because it sets different relaxation coefficients for different constraint conditions. The numerical example shows that this method can meet the requirements of convergence and precision.
(3) Live-load and prestressing effects can be included in the reasonable dead-load state of self-anchored cable-stayed-suspension bridges.
(4) The main characteristic that distinguishes self-anchored cable-stayed-suspension bridges from traditional cable-supported bridges is the intercoupling between the suspended part and the stayed part. According to the cable curve design theory for suspension bridges and the cable tension adjustment strategy of cable-stayed bridges, the proposed two-stage approach can rapidly approach the reasonable dead-load state through several iterations.

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