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The Bending-Stretching Problems of Concentrated Forces and Moments on Composite Laminate with Elliptical Inclusion

含橢圓形異質複材受集中力與彎矩作用之問題解析

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摘要

近年來，藉由類史磋的方法，針對無窮複材疊層板受到平面力與彎矩作用偶合的問題，已經獲得解析解。對於受到集中力與彎矩作用無窮板的問題也得到解決。本文將應用史磋的方式，進一步探討含橢圓形異質彈性體並且在異質內外與界面上受點力源或是差排作用條件下平面力與彎矩偶合的問題。並依據不同的集中力與彎矩作用條件來討論，求得問題的解答。

關鍵詞：集中力，異質。

Abstract

Recently, by using the Stroh-like formalism, the analytical solutions of the relations for coupled stretching-bending problems, and the concentrated forces and moment problems can be obtained for infinite composite laminates. In this paper, we will be applying the Stroh-like formalism, we can get the general solutions for the problems applied by dislocation or point forces inside, outside, or on the interface of anisotropic elliptical inclusion and matrix and coupled stretching-bending problems. And according to different the concentrated forces and moment conditions are discussed and get the solutions of the problems. we like to separate three case that inplane concentrated forces

Keywords: concentrated forces ,inclusion

1.Introduction

The problem has addressed elastic inclusions embedded in an infinite matrix and dislocations or concentrated forces at points outside, inside or on the interface of an anisotropic elliptical inclusion, a general analytical solution has been obtained (Yen, Hwu and Liang, 1995), but the cited studies mostly consider two-dimensional materials, whereas analytical solutions to problems of stretching and bending of general asymmetric composite laminates have not been found.

Solutions to problems of dislocations inside elliptical inclusions or circular isotropic matrices can be found in such studies as Dundurs and Mura (1964), Dundurs and Sendeckyj (1965), Stagni (1983) and Warren (1983). An analytical solution to the problem of concentrated forces acting on an anisotropic elastic matrix is provided in (Hwu and Ting, 1989; Hwu and Yen, 1991).

Recently, Hwu (2004) used the Stroh-like formalism, to obtain Green's function for infinite composite laminates. While two-dimensional problems are solved by analytical continuation, the use of Green's functions for solving non-hole problems may help to yield Green's functions for hole problems (Hwu, 2004). Hsieh and Hwu (2002) are obtained the analytical solutions for anisotropic plates with holes/cracks/inclusions subjected to out-of-plane bending moments. This article quotes from the cited references, to study problems of dislocations or point forces inside, outside or on the interface of anisotropic elliptical inclusions and matrices.

2. Green's Functions for Composite Laminates Problems

Green's Functions for infinite composite laminates (Hwu, 2004) are used to obtain analytical solutions for an infinite matrix that contains an anisotropic elastic elliptical inclusion, and dislocations or point forces inside, outside or on the interface of anisotropic elliptical inclusion and matrix.

In the following sections, the method of analytical continuation is applied to obtain analytical solutions for the interaction of anisotropic elliptical inclusions and dislocations or point forces. Solutions to unperturbed non-hole problems must initially be known. This section firstly considers an infinite laminate that is subjected to a concentrated force $\hat{\mathbf{f}} = \left(\hat{f}_1 , \hat{f}_2\right)$ and moment $\hat{\mathbf{m}} = \left(\hat{m}_1 , \hat{m}_2\right)$ at point $\hat{\mathbf{x}} = \left(\hat{x}_1 , \hat{x}_2\right)$. The elasticity solution of this problem can be used as a fundamental solution in the boundary element method and is generally called *Green's function*. (Hwu, 2004; Hwu and Yen, 1993). The boundary conditions in each loading case are given by

$$\oint_c d\boldsymbol{\phi}_d = \hat{\mathbf{p}} \ , \ \oint_c d\mathbf{u}_d = \hat{\mathbf{p}} \ , \ \hat{\mathbf{p}} = \left(\hat{f}_1 \ \ \hat{f}_2 \ \ \hat{m}_2 \ \ -\hat{m}_1 \right)^T \tag{2.1}$$

Through satisfaction of boundary conditions (2.1), the unknown complex function vector $\mathbf{f}$ (z) has been determined to be (Hwu, 2004)

$$\mathbf{f}(z) = <\log(z_\alpha - \hat{z}_\alpha)> \mathbf{q}_1 \tag{2.2a}$$

Where

for force $$\mathbf{q}_1 = \frac{1}{2\pi i}\mathbf{A}^T\hat{\mathbf{p}} \tag{2.2b}$$

for dislocation $\mathbf{q}_1 = \dfrac{1}{2\pi i}\mathbf{B}^T\hat{\mathbf{b}}$ (2.2c)

and $\mathbf{i}_2 = \begin{Bmatrix}0\\1\\0\\0\end{Bmatrix}$, $\mathbf{i}_3 = \begin{Bmatrix}0\\0\\1\\0\end{Bmatrix}$ (2.2d)

The angular bracket stands for the diagonal matrix whose components vary according to the subscript α, α=1,2,3,4, i.e. $< f_\alpha >= diag[f_1, f_2, f_3, f_4]$

3. Interaction of the anisotropic elliptical inclusion and dislocations or point forces

The relation about point forces or dislocation and inclusion can be divided into three parts which inside, outside or on the interface of inclusion. From some articles, the solution forms of the point forces and dislocations ,we find that the governing mathematical forms are the same. These differ only in their boundary conditions.So we can choose one to discuss point forces or dislocation alone. We will discuss dislocation in the article. And using the solutions of the anisotropic elastic containing holes, try to get the necessary answer further.

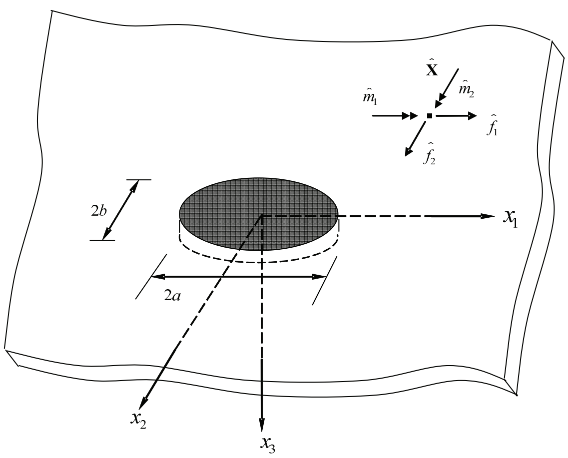


Figure1.An elliptic inclusion in laminates subjected to concentrated forces and moments

3.1 Elasticity Formulation an Elliptical Inclusion

By applying the Stroh's formalism (Ting, 1996; Stroh, 1958), a general solution satisfying these equations may be expressed as (Hwu and Yen, 1993). Consider the elliptical anisotropic inclusion imbedded in an infinite matrix, the elliptical inclusion boundary in the z-plane will be mapped to four different slanted elliptical inclusion boundary in the z_α-planes for different eigenvalues $\mu_\alpha, \alpha = 1,2,3,4$. It is not convenient to solve problems with elliptical boundary by using the argument z_α defined in $\mathbf{f}(z)$. Therefore, to treat the problems with elliptical boundary, most of the solutions shown in the literature are expressed in terms of the transformed complex variable ζ_α. These four different z_α-planes will be mapped to four different ζ_α -planes for different eigenvalues μ_α , all four different slanted elliptical inclusion boundary into the same inclusion boundary in the shape of a unit circle $|\zeta| = 1$. The relation between z_α and ζ_α is

$$z_\alpha = \frac{1}{2}\left\{(a - ib\mu_\alpha)\zeta_\alpha + (a + ib\mu_\alpha)\frac{1}{\zeta_\alpha}\right\} \text{ , } \alpha = 1,2,3,4 \quad (3.1a)$$

or inversely

$$\zeta_\alpha = \frac{z_\alpha + \sqrt{z_\alpha^2 - a^2 - b^2\mu_\alpha^2}}{a - ib\mu_\alpha} \text{ , } \alpha = 1,2,3,4 \quad (3.1b)$$

where $2a$ and $2b$ are the lengths of the major and minor axes of the ellipse, respectively. The singular point $\hat{z}_\alpha$ is the location of the dislocation or the point force. The general solution to the inclusion problems may now be written as

$$\left.\begin{aligned} \mathbf{u}_{d1} &= \mathbf{A}_1\left[\mathbf{f}_0(\zeta)+\mathbf{f}_1(\zeta)\right]+\overline{\mathbf{A}}_1\left[\overline{\mathbf{f}_0(\zeta)}+\overline{\mathbf{f}_1(\zeta)}\right] \\ \boldsymbol{\phi}_{d1} &= \mathbf{B}_1\left[\mathbf{f}_0(\zeta)+\mathbf{f}_1(\zeta)\right]+\overline{\mathbf{B}}_1\left[\overline{\mathbf{f}_0(\zeta)}+\overline{\mathbf{f}_1(\zeta)}\right] \end{aligned}\right\} \ ,\ \zeta \in S_1 \tag{3.2a}$$

and

$$\left.\begin{aligned} \mathbf{u}_{d2} &= \mathbf{A}_2\left[\mathbf{f}_0^*(\zeta)+\mathbf{f}_2(\zeta)\right]+\overline{\mathbf{A}}_1\left[\overline{\mathbf{f}_0^*(\zeta)}+\overline{\mathbf{f}_2(\zeta)}\right] \\ \boldsymbol{\phi}_{d2} &= \mathbf{B}_2\left[\mathbf{f}_0^*(\zeta)+\mathbf{f}_2(\zeta)\right]+\overline{\mathbf{B}}_1\left[\overline{\mathbf{f}_0^*(\zeta)}+\overline{\mathbf{f}_2(\zeta)}\right] \end{aligned}\right\} \ ,\ \zeta \in S_2 \tag{3.2b}$$

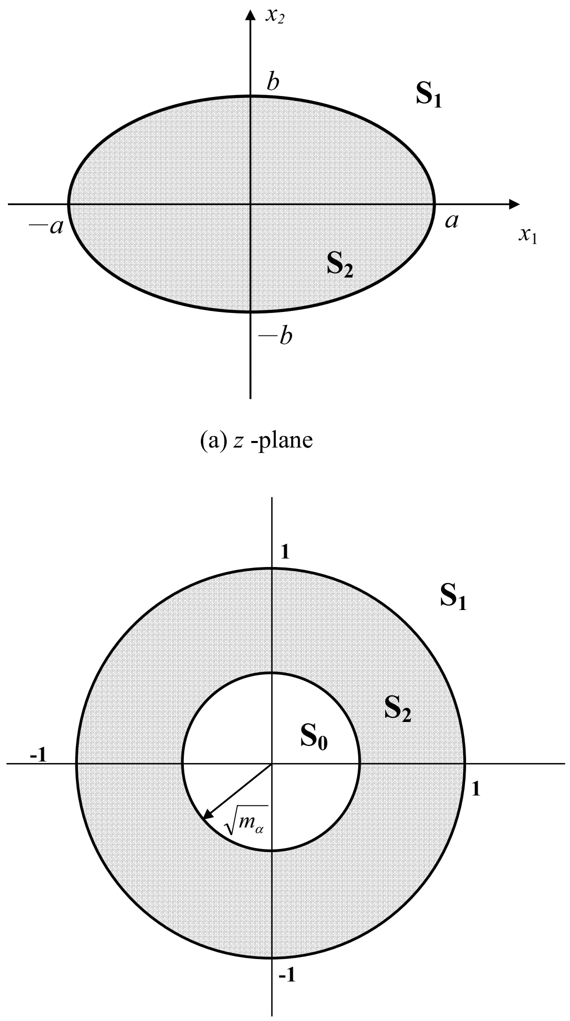
Where the subscripts 1 and 2 denote, respectively, the matrix and inclusion $\mathbf{f}_0$ and $\mathbf{f}_0^*$ represent the function associated with the singularity behavior caused by the dislocation (or point force). $\mathbf{f}_1$ (or $\mathbf{f}_2$)is the function corresponding to the flied of matrix (or inclusion) and is holomorphic in region S_1 (or S_2). S_1 and S_2 denote, respectively, the regions occupied by the matrix and inclusion.

In general, the transformation function used in Eq. (3.1) is multi-valued. However, by choosing only the mapped points outside the unit circle, the transformation function will map the region outside the elliptic inclusion onto the exterior of a unit circle and is single valued outside the inclusion. The z_α along the interface ($x_1 = a\cos\psi, x_2 = b\sin\psi$) will then be mapped onto a unit circle $\zeta_\alpha = e^{i\psi} = \sigma$. However, by choosing only the mapped points inside the unit circle, the transformation is still double-valued inside the inclusion. To have a one-to-one transformation inside the inclusion, a special choice has been made and a certain restriction should be satisfied for the function $\mathbf{f}_2(\zeta)$ and the series form expression may be written as Hwu and Yen (1993) and Lekhnitskii (1968)

$$\mathbf{f}_2(\zeta) = \sum_{k=0}^{\infty}\mathbf{c}_k\zeta^k + \sum_{k=1}^{\infty}<\gamma_\alpha^k>\mathbf{c}_k\zeta^{-k} \tag{3.3a}$$

where
$$\gamma_\alpha = \frac{a+ib\mu_\alpha^*}{a-ib\mu_\alpha^*} \tag{3.3b}$$

and μ_α^* is the material eigenvalues of the inclusion. The angular bracket stands for the diagonal matrix.



(a) z -plane

(b) ζ_α -plane

Figure 2. Mapping from (a) z-plane to(b) ζ_α -plane

3.2 Dislocation or Point Force Outside an Elliptical Inclusion

Consider an infinite matrix which includes anisotropic elastic elliptical inclusion. One dislocation with Burgers vector $\hat{\mathbf{b}}$ (or point force $\hat{\mathbf{p}}$) located at point $\hat{\mathrm{x}} = (\hat{x}_1, \hat{x}_2)$ which is outside the inclusion. If the inclusion and matrix are assumed to be perfectly bonded along the interface, the displacements and surface tractions across the interface should be continuous. That is,

$$\left.\begin{aligned}\mathbf{u}_1 &= \mathbf{u}_2\\ \phi_1 &= \phi_2\end{aligned}\right\}\ \text{along the interface} \tag{3.4}$$

An elasticity solution satisfying the dislocation singularity and the interface continuity condition has been found in [16].

We consider that the dislocation or point force is located at the infinite matrix, (3.2) will be rewritten as

$$\left.\begin{aligned}\mathrm{u}_{d1} &= \mathbf{A}_1\left[\mathbf{f}_0(\zeta)+\mathbf{f}_1(\zeta)\right]+\overline{\mathbf{A}}_1\left[\overline{\mathbf{f}_0(\zeta)}+\overline{\mathbf{f}_1(\zeta)}\right]\\ \boldsymbol{\phi}_{d1} &= \mathbf{B}_1\left[\mathbf{f}_0(\zeta)+\mathbf{f}_1(\zeta)\right]+\overline{\mathbf{B}}_1\left[\overline{\mathbf{f}_0(\zeta)}+\overline{\mathbf{f}_1(\zeta)}\right]\end{aligned}\right\}\ ,\ \zeta\in S_1 \tag{3.5a}$$

$$\left.\begin{aligned}\mathrm{u}_{d2} &= \mathbf{A}_2\left[\mathbf{f}_2(\zeta)\right]+\overline{\mathbf{A}}_1\left[\overline{\mathbf{f}_2(\zeta)}\right]\\ \boldsymbol{\phi}_{d2} &= \mathbf{B}_2\left[\mathbf{f}_2(\zeta)\right]+\overline{\mathbf{B}}_1\left[\overline{\mathbf{f}_2(\zeta)}\right]\end{aligned}\right\}\ ,\ \zeta\in S_2 \tag{3.5b}$$

Where $\mathbf{f}_0(\zeta)$ is the function that unperturbed elastic field of the matrix. $\mathbf{f}_1(\zeta)$ is the holomorphic function corresponding to the perturbed field of the matrix and will be determined through satisfaction of the boundary conditions. Divide $\mathbf{f}_0(\zeta)$ into two parts, one part is holomophic in S_1 ($\mathbf{f}_0^+(\zeta)$) and another part is holomophic in the S_2+S_0 ($\mathbf{f}_0^-(\zeta)$),Here, S_0 denotes the region inside the circle of radius $\sqrt{m_\alpha}$, $\mathbf{f}_2(\zeta)$ chose the same as $\mathbf{f}_1(\zeta)$, Express it as follows

$$\begin{gathered}\mathbf{f}_0(\zeta)=\mathbf{f}_0^+(\zeta)+\mathbf{f}_0^-(\zeta)\quad,\quad \mathbf{f}_2(\zeta)=\mathbf{f}_2^+(\zeta)+\mathbf{f}_2^-(\zeta)\\ \mathbf{f}_0^+(\zeta),\mathbf{f}_2^-(\zeta)\ \text{is holomorphic in } \mathrm{S}_1\\ \mathbf{f}_0^-(\zeta),\mathbf{f}_2^+(\zeta)\ \text{is holomorphic in } \mathrm{S}_0+\mathrm{S}_2\end{gathered} \tag{3.5c}$$

By using general solutions (3.5) ,the traction continuity condition of (3.4)

$$\begin{aligned}&\mathbf{u}_{d1}=\mathbf{u}_{d2}\\ &\quad \mathbf{A}_1\left[\mathbf{f}_0^+(\sigma)+\mathbf{f}_0^-(\sigma)+\mathbf{f}_1(\sigma)\right]+\overline{\mathbf{A}}_1\left[\overline{\mathbf{f}_0^+(\sigma)}+\overline{\mathbf{f}_0^-(\sigma)}+\overline{\mathbf{f}_1(\sigma)}\right]\\ &\quad =\mathbf{A}_2\left[\mathbf{f}_2^+(\sigma)+\mathbf{f}_2^-(\sigma)\right]+\overline{\mathbf{A}}_2\left[\overline{\mathbf{f}_2^+(\sigma)}+\overline{\mathbf{f}_2^-(\sigma)}\right]\\ &\boldsymbol{\phi}_{d1}=\boldsymbol{\phi}_{d2}\\ &\quad \mathbf{B}_1\left[\mathbf{f}_0^+(\sigma)+\mathbf{f}_0^-(\sigma)+\mathbf{f}_1(\sigma)\right]+\overline{\mathbf{B}}_1\left[\overline{\mathbf{f}_0^+(\sigma)}+\overline{\mathbf{f}_0^-(\sigma)}+\overline{\mathbf{f}_1(\sigma)}\right]\\ &\quad =\mathbf{B}_2\left[\mathbf{f}_2^+(\sigma)+\mathbf{f}_2^-(\sigma)\right]+\overline{\mathbf{B}}_2\left[\overline{\mathbf{f}_2^+(\sigma)}+\overline{\mathbf{f}_2^-(\sigma)}\right]\end{aligned} \tag{3.6}$$

One of the important properties of holomorphic functions used in the method of analytic continuation is that if $\mathbf{f}(\zeta)$ is holomorphic in S_1 （or S_2+S_0 ）,then $\overline{\mathbf{f}(1/\overline{\zeta})}$ is holomorphic in S_2+S_0 （or S_1 ）. We may introduce a function which is holomorphic in the entire domain including the interface boundary, can be rewritten as

$$\theta_1(\zeta)=\begin{cases}\mathbf{A}_1\mathbf{f}_0^+(\zeta)+\mathbf{A}_1\mathbf{f}_1(\zeta)+\overline{\mathbf{A}}_1\overline{\mathbf{f}_0^-\left(\frac{1}{\overline{\zeta}}\right)}-\mathbf{A}_2\mathbf{f}_2^-(\zeta)-\overline{\mathbf{A}}_2\overline{\mathbf{f}_2^-\left(\frac{1}{\overline{\zeta}}\right)}\ ,\ \zeta\in \mathrm{S}_1\\ -\overline{\mathbf{A}}_1\overline{\mathbf{f}_0^+\left(\frac{1}{\overline{\zeta}}\right)}-\overline{\mathbf{A}}_1\overline{\mathbf{f}_1\left(\frac{1}{\overline{\zeta}}\right)}-\mathbf{A}_1\mathbf{f}_0^-(\zeta)+\overline{\mathbf{A}}_2\overline{\mathbf{f}_2^+\left(\frac{1}{\overline{\zeta}}\right)}+\mathbf{A}_2\mathbf{f}_2^+(\zeta)\ ,\ \zeta\in \mathrm{S}_0+\mathrm{S}_2\end{cases} \tag{3.7a}$$

$$\theta_2(\zeta)=\begin{cases}\mathbf{B}_1\mathbf{f}_0^+(\zeta)+\mathbf{B}_1\mathbf{f}_1(\zeta)+\overline{\mathbf{B}}_1\overline{\mathbf{f}_0^-\left(\frac{1}{\overline{\zeta}}\right)}=\mathbf{B}_2\mathbf{f}_2^-(\zeta)+\overline{\mathbf{B}}_2\overline{\mathbf{f}_2^-\left(\frac{1}{\overline{\zeta}}\right)},\ \zeta\in \mathrm{S}_1\\ \overline{\mathbf{B}}_1\overline{\mathbf{f}_0^+\left(\frac{1}{\overline{\zeta}}\right)}+\overline{\mathbf{B}}_1\overline{\mathbf{f}_1\left(\frac{1}{\overline{\zeta}}\right)}+\mathbf{B}_1\mathbf{f}_0^-(\zeta)=\overline{\mathbf{B}}_2\overline{\mathbf{f}_2^+\left(\frac{1}{\overline{\zeta}}\right)}+\mathbf{B}_2\mathbf{f}_2^+(\zeta),\ \zeta\in \mathrm{S}_0+\mathrm{S}_2\end{cases} \tag{3.7b}$$

$\theta_1(\zeta)$ and $\theta_2(\zeta)$ are holomorphic in the whole ζ-plane including the points at infinity. By Liouville's theorem we have $\theta_1(\zeta)$ and $\theta_2(\zeta)$≡constant. However, the constant function corresponds to rigid body motion which may be neglected.

Therefore $\theta_1(\zeta)$ =0, $\theta_2(\zeta)$ =0. With this result, (3.7) can get to

$$\begin{cases}\mathbf{A}_1\mathbf{f}_0^+(\zeta)+\mathbf{A}_1\mathbf{f}_1(\zeta)+\overline{\mathbf{A}}_1\overline{\mathbf{f}_0^-\left(\frac{1}{\overline{\zeta}}\right)}=\mathbf{A}_2\mathbf{f}_2^+(\zeta)+\overline{\mathbf{A}}_2\overline{\mathbf{f}_2^-\left(\frac{1}{\overline{\zeta}}\right)}\\ \mathbf{B}_1\mathbf{f}_0^+(\zeta)+\mathbf{B}_1\mathbf{f}_1(\zeta)+\overline{\mathbf{B}}_1\overline{\mathbf{f}_0^-\left(\frac{1}{\overline{\zeta}}\right)}=\mathbf{B}_2\mathbf{f}_2^+(\zeta)+\overline{\mathbf{B}}_2\overline{\mathbf{f}_2^-\left(\frac{1}{\overline{\zeta}}\right)}\end{cases}\ ,\ \zeta\in \mathrm{S}_1 \tag{3.8a}$$

$$\begin{cases} \overline{\mathbf{A}}_1 \overline{\mathbf{f}_0^+\left(\frac{1}{\bar\zeta}\right)} + \overline{\mathbf{A}}_1 \overline{\mathbf{f}_1\left(\frac{1}{\bar\zeta}\right)} + \mathbf{A}_1\mathbf{f}_0^-(\zeta) = \overline{\mathbf{A}}_2 \overline{\mathbf{f}_2^+\left(\frac{1}{\bar\zeta}\right)} + \mathbf{A}_2\mathbf{f}_2^-(\zeta) \\ \overline{\mathbf{B}}_1 \overline{\mathbf{f}_0^+\left(\frac{1}{\bar\zeta}\right)} + \overline{\mathbf{B}}_1 \overline{\mathbf{f}_1\left(\frac{1}{\bar\zeta}\right)} + \mathbf{B}_1\mathbf{f}_0^-(\zeta) = \overline{\mathbf{B}}_2 \overline{\mathbf{f}_2^+\left(\frac{1}{\bar\zeta}\right)} + \mathbf{B}_2\mathbf{f}_2^-(\zeta) \end{cases}, \ \zeta \in S_0 + S_2 \tag{3.8b}$$

Cancellation of $\mathbf{f}_1(\zeta)$ between (3.8a) and (3.8b) leads to

$$\begin{aligned} \left(\mathbf{A}_1^{-1}\overline{\mathbf{A}}_1 - \mathbf{B}_1^{-1}\overline{\mathbf{B}}_1\right)\overline{\mathbf{f}_0^-(1/\bar\zeta)} &= \left(\mathbf{A}_1^{-1}\overline{\mathbf{A}}_2 - \mathbf{B}_1^{-1}\overline{\mathbf{B}}_2\right)\overline{\mathbf{f}_2^-(1/\bar\zeta)} \\ &\quad + \left(\mathbf{A}_1^{-1}\mathbf{A}_2 - \mathbf{B}_1^{-1}\mathbf{B}_2\right)\mathbf{f}_2^+(\zeta), \ \zeta \in S_1 \\ \left(\overline{\mathbf{A}}_1^{-1}\mathbf{A}_1 - \overline{\mathbf{B}}_1^{-1}\mathbf{B}_1\right)\mathbf{f}_0^-(\zeta) &= \left(\overline{\mathbf{A}}_1^{-1}\mathbf{A}_2 - \overline{\mathbf{B}}_1^{-1}\mathbf{B}_2\right)\mathbf{f}_2^-(\zeta) \\ &\quad + \left(\overline{\mathbf{A}}_1^{-1}\overline{\mathbf{A}}_2 - \overline{\mathbf{B}}_1^{-1}\overline{\mathbf{B}}_2\right)\overline{\mathbf{f}_2^+(1/\bar\zeta)}, \ \zeta \in S_0 + S_2 \end{aligned} \tag{3.9}$$

we can have to put in order from $(3.9)_2$

$$\begin{aligned} -i\mathbf{A}_1^{-T}\mathbf{f}_0^-(\zeta) &= \left(\overline{\mathbf{M}}_1 + \mathbf{M}_2\right)\mathbf{A}_2\mathbf{f}_2^-(\zeta) + \left(\overline{\mathbf{M}}_1 - \overline{\mathbf{M}}_2\right)\overline{\mathbf{A}}_2\overline{\mathbf{f}_2^+(1/\bar\zeta)} \\ \mathbf{M}_k &= -i\mathbf{B}_k\mathbf{A}_k^{-1}, \quad k = 1,2 \end{aligned} \tag{3.10}$$

Because $\mathbf{f}_0(\zeta)$ has already known, from (3.10) can get to solution for $\mathbf{f}_2(\zeta)$. Using the conditions of (3.4) will be tried to get the function of $\mathbf{f}_1(\zeta)$

We discuss the boundary conditions of different load situations (2.1), using the Green's functions of composite laminates problems (2.2) , we can solve the general solutions of disloaction outside the elliptical inclusion,

We consider the matrix function including singularity points, received the concentrated forces and concentrated moments in first and second coordinate axis. We may choose $\mathbf{f}_0(\zeta)$ as

$$\mathbf{f}_0(z) = <\log(z_\alpha - \hat{z}_\alpha)> \mathbf{q}_1 \tag{2.2a}$$

From (3.1) we know that

$$\mathbf{f}_0(\zeta) = <\log(\zeta_\alpha - \hat\zeta_\alpha) + \log\left(1 - \frac{\gamma_\alpha}{\zeta_\alpha\hat\zeta_\alpha}\right) + \log c_\alpha > \mathbf{q}_1 \tag{3.11}$$

As a result of $\mathbf{f}_0^+(\zeta)$ is holomorphic in S_1 and $\mathbf{f}_0^-(\zeta)$ is holomorphic in S_2, we get

$$\begin{aligned} \mathbf{f}_0^+(\zeta) &= <\log\left(1 - \frac{\gamma_\alpha}{\zeta_\alpha\hat\zeta_\alpha}\right)> \mathbf{q}_1, \\ \mathbf{f}_0^-(\zeta) &= <\log(\zeta_\alpha - \hat\zeta_\alpha)> \mathbf{q}_1 \end{aligned} \tag{3.12}$$

The constant term $\log c_\alpha$ has been neglected since it corresponds to rigid body motion and has no contribution to the deformation.

For easy to calculate, it can be represented by series expansion,

$$\begin{aligned} \mathbf{f}_0^-(\zeta) &= <\log(\zeta_\alpha - \hat\zeta_\alpha)> \mathbf{q}_1 = \sum_{k=1}^{\infty}\mathbf{e}_k\zeta^k \\ \mathbf{e}_k &= \frac{-1}{k}\hat\zeta_\alpha^{-k}\mathbf{q}_1 \end{aligned} \tag{3.13}$$

$$\mathbf{f}_2^+(\zeta) = \sum_{k=1}^{\infty} <\gamma_\alpha^k> \mathbf{c}_k\zeta^{-k}, \quad \mathbf{f}_2^-(\zeta) = \sum_{k=1}^{\infty}\mathbf{c}_k\zeta^k \tag{3.14}$$

Take (3.13) and (3.14) into (3.10) we can get

$$\mathbf{c}_k = \left\{\mathbf{G}_0 - \overline{\mathbf{G}}_k\overline{\mathbf{G}}_0^{-1}\mathbf{G}_k\right\}^{-1}\left\{\mathbf{t}_k - \overline{\mathbf{G}}_k\overline{\mathbf{G}}_0^{-1}\bar{\mathbf{t}}_k\right\} \tag{3.15a}$$

Where

$$\mathbf{G}_0 = \left(\overline{\mathbf{M}}_1 + \mathbf{M}_2\right)\mathbf{A}_2, \ \overline{\mathbf{G}}_k = \left(\overline{\mathbf{M}}_1 - \overline{\mathbf{M}}_2\right)\overline{\mathbf{A}}_2 <\bar\gamma_\alpha^k>, \quad \mathbf{t}_k = -i\mathbf{A}_1^{-T}\mathbf{e}_k \tag{3.15b}$$

Having the solution of $\mathbf{c}_k$, function $\mathbf{f}_1(\zeta)$ can now be obtained from (3.8)

$$\begin{aligned} \mathbf{f}_1(\zeta) &= -<\log\left(1 - \frac{\gamma_\alpha}{\zeta_\alpha\hat\zeta_\alpha}\right)> \mathbf{q}_1 \\ &\quad - \mathbf{A}_1^{-1}\sum_{k=1}^{\infty}\left[\overline{\mathbf{A}}_1\bar{\mathbf{e}}_k - \mathbf{A}_2 <\gamma_\alpha^k> \mathbf{c}_k - \overline{\mathbf{A}}_2\bar{\mathbf{c}}_k\right]\zeta^{-k} \\ \text{or} \quad \mathbf{f}_1(\zeta) &= -<\log\left(1 - \frac{\gamma_\alpha}{\zeta_\alpha\hat\zeta_\alpha}\right)> \mathbf{q}_1 \\ &\quad - \mathbf{B}_1^{-1}\sum_{k=1}^{\infty}\left[\overline{\mathbf{B}}_1\bar{\mathbf{e}}_k - \mathbf{B}_2 <\gamma_\alpha^k> \mathbf{c}_k - \overline{\mathbf{B}}_2\bar{\mathbf{c}}_k\right]\zeta^{-k} \end{aligned} \tag{3.16}$$

3.3 Dislocation or Point Force Inside an Elliptical Inclusion

Consider the dislocation (or point force) is located at the point which is inside the inclusion. Because singularity points lies on the inculsion will cause holomorphic continuation conditions to produce the question. So must consider getting rid of the singularity points on the choice of function in inclusion. And the choice of function in matrix must correlate with inclusion, even the same as singularity function.

So we rewrite (3.2) of matrix and inclusion function as

$$\left.\begin{aligned} \mathbf{u}_{d1} &= \mathbf{A}_1\left[\mathbf{f}_0(\zeta)+\mathbf{f}_1(\zeta)\right]+\overline{\mathbf{A}}_1\left[\overline{\mathbf{f}_0(\zeta)}+\overline{\mathbf{f}_1(\zeta)}\right] \\ \boldsymbol{\phi}_{d1} &= \mathbf{B}_1\left[\mathbf{f}_0(\zeta)+\mathbf{f}_1(\zeta)\right]+\overline{\mathbf{B}}_1\left[\overline{\mathbf{f}_0(\zeta)}+\overline{\mathbf{f}_1(\zeta)}\right] \end{aligned}\right\} \quad ,\ \zeta\in S_1 \tag{3.17a}$$

$$\left.\begin{aligned} \mathbf{u}_{d2} &= \mathbf{A}_2\left[\mathbf{f}_0^*(\zeta)+\mathbf{f}_2(\zeta)\right]+\overline{\mathbf{A}}_1\left[\overline{\mathbf{f}_0^*(\zeta)}+\overline{\mathbf{f}_2(\zeta)}\right] \\ \boldsymbol{\phi}_{d2} &= \mathbf{B}_2\left[\mathbf{f}_0^*(\zeta)+\mathbf{f}_2(\zeta)\right]+\overline{\mathbf{B}}_1\left[\overline{\mathbf{f}_0^*(\zeta)}+\overline{\mathbf{f}_2(\zeta)}\right] \end{aligned}\right\} \quad ,\ \zeta\in S_2 \tag{3.17b}$$

where
$$\begin{aligned} \mathbf{f}_2(\zeta) &= \mathbf{f}_2^+(\zeta)+\mathbf{f}_2^-(\zeta) \\ \mathbf{f}_0^*(\zeta) &= \mathbf{f}_0^{*+}(\zeta)+\mathbf{f}_0^{*-}(\zeta)+\mathbf{f}_p^*(\zeta) \\ \mathbf{f}_0(\zeta) &= \mathbf{f}_p(\zeta) \end{aligned} \tag{3.17c}$$

$\mathbf{f}_0(\zeta)$ and $\mathbf{f}_0^*(\zeta)$ represent functions associated with the singular behavior caused by the point forces or dislocation; $\mathbf{f}_p^*(\zeta)$ describes a singular phenomenon in the inclusion, and $\mathbf{f}_p(\zeta)$ is a singular phenomenon in the matrix

Satisfying the point forces or dislocation singularity conditions and applying the interface continuity condition (3.4), we can get

$$\begin{aligned} &\mathbf{u}_{d1}=\mathbf{u}_{d2} \\ &\mathbf{A}_1\left[\mathbf{f}_0(\sigma)+\mathbf{f}_1(\sigma)\right]+\overline{\mathbf{A}}_1\left[\overline{\mathbf{f}_0(\sigma)}+\overline{\mathbf{f}_1(\sigma)}\right] \\ &\quad=\mathbf{A}_2\left[\mathbf{f}_0^{*+}(\sigma)+\mathbf{f}_0^{*-}(\sigma)+\mathbf{f}_p^*(\sigma)+\mathbf{f}_2^+(\sigma)+\mathbf{f}_2^-(\sigma)\right] \\ &\qquad+\overline{\mathbf{A}}_2\left[\overline{\mathbf{f}_0^{*+}(\sigma)}+\overline{\mathbf{f}_0^{*-}(\sigma)}+\overline{\mathbf{f}_p^*(\sigma)}+\overline{\mathbf{f}_2^+(\sigma)}+\overline{\mathbf{f}_2^-(\sigma)}\right] \\ &\boldsymbol{\phi}_{d1}=\boldsymbol{\phi}_{d2} \\ &\mathbf{B}_1\left[\mathbf{f}_0(\sigma)+\mathbf{f}_1(\sigma)\right]+\overline{\mathbf{B}}_1\left[\overline{\mathbf{f}_0(\sigma)}+\overline{\mathbf{f}_1(\sigma)}\right] \\ &\quad=\mathbf{B}_2\left[\mathbf{f}_0^{*+}(\sigma)+\mathbf{f}_0^{*-}(\sigma)+\mathbf{f}_p^*(\sigma)+\mathbf{f}_2^+(\sigma)+\mathbf{f}_2^+(\sigma)\right] \\ &\qquad+\overline{\mathbf{B}}_2\left[\overline{\mathbf{f}_0^{*+}(\sigma)}+\overline{\mathbf{f}_0^{*-}(\sigma)}+\overline{\mathbf{f}_p^*(\sigma)}+\overline{\mathbf{f}_2^+(\sigma)}+\overline{\mathbf{f}_2^-(\sigma)}\right] \end{aligned} \tag{3.18}$$

After comparing, can take the following equality , and therefore equality tries to get the solutions of singularity functions.

$$\begin{aligned} \mathbf{A}_1\mathbf{f}_0(\sigma)+\overline{\mathbf{A}}_1\overline{\mathbf{f}_0(\sigma)} &= \mathbf{A}_1\mathbf{f}_p(\sigma)+\overline{\mathbf{A}}_1\overline{\mathbf{f}_p(\sigma)} = \mathbf{A}_2\mathbf{f}_p^*(\sigma)+\overline{\mathbf{A}}_2\overline{\mathbf{f}_p^*(\sigma)} \\ \mathbf{B}_1\mathbf{f}_0(\sigma)+\overline{\mathbf{B}}_1\overline{\mathbf{f}_0(\sigma)} &= \mathbf{B}_1\mathbf{f}_p(\sigma)+\overline{\mathbf{B}}_1\overline{\mathbf{f}_p(\sigma)} = \mathbf{B}_2\mathbf{f}_p^*(\sigma)+\overline{\mathbf{B}}_2\overline{\mathbf{f}_p^*(\sigma)} \end{aligned} \tag{3.19}$$

Later used analytical continuation method to have

$$\begin{cases} \mathbf{A}_1\mathbf{f}_1(\zeta)=\mathbf{A}_2\mathbf{f}_0^{*+}(\zeta)+\overline{\mathbf{A}}_2\overline{\mathbf{f}_0^{*-}\left(\frac{1}{\bar{\zeta}}\right)}+\mathbf{A}_2\mathbf{f}_2^+(\zeta)+\overline{\mathbf{A}}_2\overline{\mathbf{f}_2^-\left(\frac{1}{\bar{\zeta}}\right)} \ ,\ \zeta\in S_1 \\ \overline{\mathbf{A}}_1\overline{\mathbf{f}_1\left(\frac{1}{\bar{\zeta}}\right)}=\overline{\mathbf{A}}_2\overline{\mathbf{f}_0^{*+}\left(\frac{1}{\bar{\zeta}}\right)}+\mathbf{A}_2\mathbf{f}_0^{*-}(\zeta)+\overline{\mathbf{A}}_2\overline{\mathbf{f}_2^+\left(\frac{1}{\bar{\zeta}}\right)}+\mathbf{A}_2\mathbf{f}_2^-(\zeta)\ ,\ \zeta\in S_0+S_2 \end{cases} \tag{3.20a}$$

$$\begin{cases} \mathbf{B}_1\mathbf{f}_1(\zeta)=\mathbf{B}_2\mathbf{f}_0^{*+}(\zeta)+\overline{\mathbf{B}}_2\overline{\mathbf{f}_0^{*-}\left(\frac{1}{\bar{\zeta}}\right)}+\mathbf{B}_2\mathbf{f}_2^+(\zeta)+\overline{\mathbf{B}}_2\overline{\mathbf{f}_2^-\left(\frac{1}{\bar{\zeta}}\right)},\ \zeta\in S_1 \\ \overline{\mathbf{B}}_1\overline{\mathbf{f}_1\left(\frac{1}{\bar{\zeta}}\right)}=\overline{\mathbf{B}}_2\overline{\mathbf{f}_0^{*+}\left(\frac{1}{\bar{\zeta}}\right)}+\mathbf{B}_2\mathbf{f}_0^{*-}(\zeta)+\overline{\mathbf{B}}_2\overline{\mathbf{f}_2^+\left(\frac{1}{\bar{\zeta}}\right)}+\mathbf{B}_2\mathbf{f}_2^-(\zeta),\zeta\in S_0+S_2 \end{cases} \tag{3.20b}$$

Cancellation of $\mathbf{f}_1(\zeta)$ between (3.20a) and (3.20b) leads to

$$\begin{aligned} &\left(\mathbf{A}_1^{-1}\mathbf{A}_2-\mathbf{B}_1^{-1}\mathbf{B}_2\right)\mathbf{f}_0^{*+}(\zeta)+\left(\mathbf{A}_1^{-1}\overline{\mathbf{A}}_2-\mathbf{B}_1^{-1}\overline{\mathbf{B}}_2\right)\overline{\mathbf{f}_0^{*-}(1/\bar{\zeta})} \\ &\qquad=-\left(\mathbf{A}_1^{-1}\mathbf{A}_2-\mathbf{B}_1^{-1}\mathbf{B}_2\right)\mathbf{f}_2^+(\zeta)-\left(\mathbf{A}_1^{-1}\overline{\mathbf{A}}_2-\mathbf{B}_1^{-1}\overline{\mathbf{B}}_2\right)\overline{\mathbf{f}_2^-(1/\bar{\zeta})}\ ,\zeta\in S_1 \\ &\left(\overline{\mathbf{A}}_1^{-1}\overline{\mathbf{A}}_2-\overline{\mathbf{B}}_1^{-1}\overline{\mathbf{B}}_2\right)\overline{\mathbf{f}_0^{*+}(1/\bar{\zeta})}+\left(\overline{\mathbf{A}}_1^{-1}\mathbf{A}_2-\overline{\mathbf{B}}_1^{-1}\mathbf{B}_2\right)\mathbf{f}_0^{*-}(\zeta) \\ &\qquad=-\left(\overline{\mathbf{A}}_1^{-1}\overline{\mathbf{A}}_2-\overline{\mathbf{B}}_1^{-1}\overline{\mathbf{B}}_2\right)\overline{\mathbf{f}_2^+(1/\bar{\zeta})}-\left(\overline{\mathbf{A}}_1^{-1}\mathbf{A}_2-\overline{\mathbf{B}}_1^{-1}\mathbf{B}_2\right)\mathbf{f}_2^-(\zeta)\ ,\zeta\in S_0+S_2 \end{aligned} \tag{3.21}$$

we can have to put in order from $(3.21)_2$

$$\begin{aligned} -\left(\overline{\mathbf{M}}_1-\overline{\mathbf{M}}_2\right)\overline{\mathbf{A}}_2\overline{\mathbf{f}_0^{*+}(1/\bar{\zeta})}-\left(\overline{\mathbf{M}}_1+\mathbf{M}_2\right)\mathbf{A}_2\mathbf{f}_0^{*-}(\zeta) \\ =\left(\overline{\mathbf{M}}_1+\mathbf{M}_2\right)\mathbf{A}_2\mathbf{f}_2^-(\zeta)+\left(\overline{\mathbf{M}}_1-\overline{\mathbf{M}}_2\right)\overline{\mathbf{A}}_2\overline{\mathbf{f}_2^+(1/\bar{\zeta})} \end{aligned} \tag{3.22}$$

Because $\mathbf{f}_0^*(\zeta)$ has already known, From (3.22) can get to solution for $\mathbf{f}_2(\zeta)$. Using the conditions of (3.20) will be tried to get the function of $\mathbf{f}_1(\zeta)$

We discuss the boundary conditions of different load situations (2.1), using the Green's functions of composite laminates problems (2.2) , we can solve the general solutions of disloaction inside the elliptical inclusion.

When dislocations lies in elliptical inclusion, the influence that the singularity phenomenon of dislocations causes, will happen on elliptical inclusion. By (3.17b) we will be divided into the perturbed and unperturbed function of inclusion. Where $\mathbf{f}_0^*(\zeta)$ is the function that unperturbed elastic field of the inclusion. $\mathbf{f}_2(\zeta)$ is the holomorphic function corresponding to the perturbed field of the inclusion. In order to satsify the dislocation singularity and interface continuity conditions, we will choose $\mathbf{f}_0^*(\zeta)$ as the form of (2.2a) and change parameter z_α into ζ_α. Because consider that must holomorphic inside on S_2, function is expressed as follows

$$\mathbf{f}_0^*(\zeta) = <\log\left(\zeta_\alpha - \hat{\zeta}_\alpha\right) + \log\left(1 - \frac{\gamma_\alpha}{\zeta_\alpha \hat{\zeta}_\alpha}\right) + \log c_\alpha > \mathbf{q}_1 \tag{3.23a}$$

where

$$\mathbf{q}_1 = \begin{cases} \dfrac{1}{2\pi i}\mathbf{A}_2^T\hat{\mathbf{p}} & , \text{ for point forces} \\ \dfrac{1}{2\pi i}\mathbf{B}_2^T\hat{\mathbf{b}} & , \text{ for dislocations} \end{cases} \tag{3.23b}$$

Known that $\mathbf{f}_0^*(\zeta)$ has singularity phenomenon by (3.17c), among $\mathbf{f}_0^{*+}(\zeta)$ is holomorphic in S_1 and $\mathbf{f}_0^{*-}(\zeta)$ is holomorphic in S_2, so (3.23a) make into

$$\begin{aligned} &\mathbf{f}_0^{*+}(\zeta) = <\log\left(1 - \frac{\gamma_\alpha}{\zeta_\alpha \hat{\zeta}_\alpha}\right) + \log\left(1 - \frac{\hat{\zeta}_\alpha}{\zeta_\alpha}\right) + \log c_\alpha > \mathbf{q}_1 \\ &\mathbf{f}_0^{*+}(\zeta) = \mathbf{0} \\ &\mathbf{f}_p^*(\zeta) = <\log \zeta_\alpha > \mathbf{q}_1 \end{aligned} \tag{3.24}$$

Because of (3.24) can set matrix function as

$$\mathbf{f}_0(\zeta) = <\log \zeta_\alpha > \mathbf{d} \tag{3.25}$$

substituting $(3.24)_3$ and (3.25) into (3.19) we can get

$$\mathbf{d} = \begin{cases} \dfrac{1}{2\pi i}\mathbf{A}_1^T\hat{\mathbf{p}} & , \text{ for point forces} \\ \dfrac{1}{2\pi i}\mathbf{B}_1^T\hat{\mathbf{b}} & , \text{ for dislocation} \end{cases} \tag{3.26}$$

Function must be analyzed inside on the domain, So choose function as follows

$$\begin{aligned} &\mathbf{f}_0^{*+}(\zeta) = \sum_{k=1}^{\infty}\mathbf{e}_k\zeta^{-k} \quad , \quad \mathbf{f}_0^{*-}(\zeta) = \sum_{k=1}^{\infty}\mathbf{d}_k\zeta^{k} \\ &\mathbf{f}_2^{+}(\zeta) = \sum_{k=1}^{\infty}<\gamma_\alpha^k>\mathbf{c}_k\zeta^{-k} \quad , \quad \mathbf{f}_2^{-}(\zeta) = \sum_{k=1}^{\infty}\mathbf{c}_k\zeta^{k} \end{aligned} \tag{3.27}$$

Substituting (3.27) into (3.22) we get

$$\begin{aligned} &\mathbf{c}_k = \left\{\mathbf{G}_0 - \overline{\mathbf{G}}_k\overline{\mathbf{G}}_0^{-1}\mathbf{G}_k\right\}^{-1}\left\{\mathbf{t}_k - \overline{\mathbf{G}}_k\overline{\mathbf{G}}_0^{-1}\overline{\mathbf{t}}_k\right\} \\ &\mathbf{G}_0 = \left(\overline{\mathbf{M}}_1 + \mathbf{M}_2\right)\mathbf{A}_2 \quad , \quad \overline{\mathbf{G}}_k = \left(\overline{\mathbf{M}}_1 - \overline{\mathbf{M}}_2\right)\overline{\mathbf{A}}_2 < \bar{\gamma}_\alpha^k > \\ &\mathbf{t}_k = -\left(\overline{\mathbf{M}}_1 - \overline{\mathbf{M}}_2\right)\overline{\mathbf{A}}_2\bar{\mathbf{e}}_k - \left(\overline{\mathbf{M}}_1 + \mathbf{M}_2\right)\mathbf{A}_2\mathbf{d}_k \end{aligned} \tag{3.28}$$

using series expansion to substitute for the (3.24), we can get

$$\begin{aligned} \mathbf{f}_0^{*+}(\zeta) &= <\log\left(1 - \frac{\gamma_\alpha}{\zeta_\alpha \hat{\zeta}_\alpha}\right) + \log\left(1 - \frac{\hat{\zeta}_\alpha}{\zeta_\alpha}\right) + \log c_\alpha > \mathbf{q}_1 \\ &= \sum_{k=1}^{\infty}\mathbf{e}_k\zeta_\alpha^{-k} \\ \mathbf{f}_0^{*+}(\zeta) &= \mathbf{0} = \sum_{k=1}^{\infty}\mathbf{d}_k\zeta_\alpha^{k} \end{aligned}$$

$$\mathbf{e}_k = \frac{-1}{k}\left(\hat{\zeta}_\alpha^k + \left(\frac{\gamma_\alpha}{\hat{\zeta}_\alpha}\right)^k\right)\mathbf{q}_1 \quad , \quad \mathbf{d}_k = 0 \tag{3.29}$$

Substituting (3.29) and (3.28) into (3.20) we can get solutions for $\mathbf{f}_1(\zeta)$.

$$\begin{aligned} \mathbf{f}_1(\zeta) = \mathbf{A}_1^{-1}\mathbf{A}_2 &<\log\left(1 - \frac{\gamma_\alpha}{\zeta_\alpha \hat{\zeta}_\alpha}\right) + \log\left(1 - \frac{\hat{\zeta}_\alpha}{\zeta_\alpha}\right) + \log c_\alpha > \mathbf{q}_1 \\ &+ \sum_{k=1}^{\infty}\mathbf{A}_1^{-1}\left[\mathbf{A}_2 < \gamma_\alpha^k > \mathbf{c}_k + \overline{\mathbf{A}}_2\bar{\mathbf{c}}_k\right]\zeta^{-k} \end{aligned}$$

or

$$\begin{aligned} \mathbf{f}_1(\zeta) = \mathbf{B}_1^{-1}\mathbf{B}_2 &<\log\left(1 - \frac{\gamma_\alpha}{\zeta_\alpha \hat{\zeta}_\alpha}\right) + \log\left(1 - \frac{\hat{\zeta}_\alpha}{\zeta_\alpha}\right) + \log c_\alpha > \mathbf{q}_1 \\ &+ \sum_{k=1}^{\infty}\mathbf{B}_1^{-1}\left[\mathbf{B}_2 < \gamma_\alpha^k > \mathbf{c}_k + \overline{\mathbf{B}}_2\bar{\mathbf{c}}_k\right]\zeta^{-k} \end{aligned} \tag{3.30}$$

3.4 Dislocation or Point Force on the Interface Between Inclusion and Matrix

Sections 3.2 and 3.3 explore a singular point (point force or dislocation) on the matrix or inclusion. This study further investigates singular point situations on the inclusion and matrix interface that were considered in (Yen, W. J., and Hwu, C., and Liang, Y. K., 1995) When singularity behaviors are exhibited at a point on the interface, the choice of $\mathbf{f}_0(\zeta)$ and $\mathbf{f}_0^*(\zeta)$ is not as clear as in Sections 3.2 and 3.3, because the singular point is covered by both the inclusion and the matrix. Hence, the singular term between the matrix and the inclusion in the same as Section 3.3 must be considered. A function (3.17) is chosen. Applying the singularity boundary conditions ,we can get solutions

for dislocation singularity $\left.\begin{array}{l}\oint d\boldsymbol{\phi}_d = 0\\ \oint d\mathbf{u}_d = \hat{\mathbf{b}}\end{array}\right\}$ (3.31a)

for point force singularity $\left.\begin{array}{l}\oint d\boldsymbol{\phi}_d = \hat{\mathbf{p}}\\ \oint d\mathbf{u}_d = 0\end{array}\right\}$ (3.31b)

Continuity conditions are then applied to determine the unknown coefficient, which is substituted into the function to yield the unknown function $\mathbf{f}_2(\zeta)$

We discuss the boundary conditions of different load situations (2.1), by (2.2) we can get the general solutions of disloaction on the interface of anisotropic elliptical inclusion and matrix.

Because singular point lies on the interface of matrix and inclusion at the same time, we can look for the forms of $\mathbf{f}_0(\zeta)$ and $\mathbf{f}_0^*(\zeta)$ from general solutions of hole and elliptical plate problems , choose as follows

$$\mathbf{f}_0(\zeta) = <\log(\zeta_\alpha - \hat{\zeta}_\alpha)> \mathbf{q}_1 + \log\left(\zeta_\alpha^{-1} - \bar{\hat{\zeta}}_\alpha\right) > \mathbf{q}_1'$$

$$\mathbf{f}_0^*(\zeta) = <\log(\zeta_\alpha - \hat{\zeta}_\alpha) + \log\left(1 - \frac{\gamma_\alpha}{\zeta_\alpha \hat{\zeta}_\alpha}\right) + \log c_\alpha > \mathbf{q}_2$$

(3.32)

where $\mathbf{q}_1, \mathbf{q}_1'$, and $\mathbf{q}_2$ are the unknown coefficients to be determined.

General like section 3.3, in order to satisfy every region holomorphic, we are divided (3.32) into and influenced by singularity phenomenon and free of singularity phenomenon two parts, rewrite it as follows:

$$\mathbf{f}_0^{*+}(\zeta) = <\log\left(1 - \frac{\gamma_\alpha}{\zeta_\alpha \hat{\zeta}_\alpha}\right) > \mathbf{q}_2 = \sum_{k=1}^{\infty} \mathbf{e}_k \zeta^{-k}$$

$$\mathbf{f}_0^{*-}(\zeta) = \sum_{k=1}^{\infty} \mathbf{d}_k \zeta^k = \mathbf{0}$$

$$\mathbf{f}_0(\zeta) = \mathbf{f}_p(\zeta) = <\log(\zeta_\alpha - \hat{\zeta}_\alpha)> \mathbf{q}_1 + \log\left(\zeta_\alpha^{-1} - \bar{\hat{\zeta}}_\alpha\right) > \mathbf{q}_1'$$

$$\mathbf{f}_p^*(\zeta) = <\log(\zeta_\alpha - \hat{\zeta}_\alpha)> \mathbf{q}_2$$

$$\mathbf{e}_k = <\frac{-1}{k}\left(\frac{\gamma_\alpha}{\hat{\zeta}_\alpha}\right)^k > \mathbf{q}_2 \ , \ \mathbf{d}_k = \mathbf{0}$$

(3.33)

Like (3.19) comparing singular term and can have

$$\mathbf{A}_1\mathbf{q}_1 + \mathbf{A}_1\mathbf{q}' - \mathbf{A}_2\mathbf{q}_2 + \bar{\mathbf{A}}_1\bar{\mathbf{q}}_1 + \bar{\mathbf{A}}_1\bar{\mathbf{q}}_1' - \bar{\mathbf{A}}_2\bar{\mathbf{q}}_2 = 0$$

$$\mathbf{A}_1\mathbf{q}_1 - \mathbf{A}_1\mathbf{q}' - \mathbf{A}_2\mathbf{q}_2 - \bar{\mathbf{A}}_1\bar{\mathbf{q}}_1 + \bar{\mathbf{A}}_1\bar{\mathbf{q}}_1' + \bar{\mathbf{A}}_2\bar{\mathbf{q}}_2 = 0$$

(3.34a)

$$\mathbf{B}_1\mathbf{q}_1 + \mathbf{B}_1\mathbf{q}' - \mathbf{B}_2\mathbf{q}_2 + \bar{\mathbf{B}}_1\bar{\mathbf{q}}_1 + \bar{\mathbf{B}}_1\bar{\mathbf{q}}_1' - \bar{\mathbf{B}}_2\bar{\mathbf{q}}_2 = 0$$

$$\mathbf{B}_1\mathbf{q}_1 - \mathbf{B}_1\mathbf{q}' - \mathbf{B}_2\mathbf{q}_2 - \bar{\mathbf{B}}_1\bar{\mathbf{q}}_1 + \bar{\mathbf{B}}_1\bar{\mathbf{q}}_1' + \bar{\mathbf{B}}_2\bar{\mathbf{q}}_2 = 0$$

(3.34b)

Consider Eq.(3.31), the mapped points around $\hat{\mathbf{x}}$ may be expressed as $\zeta_\alpha = \hat{\zeta}_\alpha + \rho e^{i\theta}$, $\rho \to 0$ where θ starts from the line tangent to the interface. Hence, the closed integrals may be expressed by $\oint d\boldsymbol{\phi}_d = \boldsymbol{\phi}_{d1}(\pi) - \boldsymbol{\phi}_{d1}(0) + \boldsymbol{\phi}_{d2}(2\pi) - \boldsymbol{\phi}_{d2}(\pi)$ and $\oint d\mathbf{u}_d = \mathbf{u}_{d1}(\pi) - \mathbf{u}_{d1}(0) + \mathbf{u}_{d2}(2\pi) - \mathbf{u}_{d2}(\pi)$.

Subtituting (3.32) and (3.3a) into (3.2), the equilibrium and the singularity conditions shown in (3.31) now provide

$$\mathbf{A}_1\mathbf{q}_1+\mathbf{A}_1\mathbf{q}'+\mathbf{A}_2\mathbf{q}_2-\overline{\mathbf{A}}_1\overline{\mathbf{q}}_1-\overline{\mathbf{A}}_1\overline{\mathbf{q}}_1'-\overline{\mathbf{A}}_2\overline{\mathbf{q}}_2=\frac{1}{\pi i}\hat{\mathbf{b}}$$
$$\mathbf{B}_1\mathbf{q}_1+\mathbf{B}_1\mathbf{q}'+\mathbf{B}_2\mathbf{q}_2-\mathbf{B}_1\overline{\mathbf{q}}_1-\overline{\mathbf{B}}_1\overline{\mathbf{q}}_1'-\overline{\mathbf{B}}_2\overline{\mathbf{q}}_2=0 \tag{3.35a}$$

or

$$\mathbf{A}_1\mathbf{q}_1+\mathbf{A}_1\mathbf{q}'+\mathbf{A}_2\mathbf{q}_2-\overline{\mathbf{A}}_1\overline{\mathbf{q}}_1-\overline{\mathbf{A}}_1\overline{\mathbf{q}}_1'-\overline{\mathbf{A}}_2\overline{\mathbf{q}}_2=0$$
$$\mathbf{B}_1\mathbf{q}_1+\mathbf{B}_1\mathbf{q}'+\mathbf{B}_2\mathbf{q}_2-\mathbf{B}_1\overline{\mathbf{q}}_1-\overline{\mathbf{B}}_1\overline{\mathbf{q}}_1'-\overline{\mathbf{B}}_2\overline{\mathbf{q}}_2=\frac{1}{\pi i}\hat{\mathbf{p}} \tag{3.35b}$$

By (3.34) and (3.35) can unknown number $\mathbf{q}_1, \mathbf{q}_1'$, and $\mathbf{q}_2$ is solved out

for dislocation

$$\mathbf{q}_1=\frac{1}{2\pi i}\mathbf{B}_1^T\hat{\mathbf{b}}$$
$$\mathbf{q}_1'=\frac{1}{2\pi i}\left(\overline{\mathbf{A}}_2^{-1}\mathbf{A}_1-\overline{\mathbf{B}}_2^{-1}\mathbf{B}_1\right)^{-1}\left(\overline{\mathbf{A}}_2^{-1}\overline{\mathbf{A}}_1-\overline{\mathbf{B}}_2^{-1}\overline{\mathbf{B}}_1\right)\mathbf{B}_1^T\hat{\mathbf{b}}$$
$$\mathbf{q}_2=\frac{1}{2\pi i}\left(\overline{\mathbf{A}}_1^{-1}\mathbf{A}_2-\overline{\mathbf{B}}_1^{-1}\mathbf{B}_2\right)^{-1}\left(\overline{\mathbf{A}}_1^{-1}\mathbf{A}_1-\overline{\mathbf{B}}_1^{-1}\mathbf{B}_1\right)\mathbf{B}_1^T\hat{\mathbf{b}} \tag{3.36a}$$

for point forces

$$\mathbf{q}_1=\frac{1}{2\pi i}\mathbf{A}_1^T\hat{\mathbf{p}}$$
$$\mathbf{q}_1'=\frac{1}{2\pi i}\left(\overline{\mathbf{A}}_2^{-1}\mathbf{A}_1-\overline{\mathbf{B}}_2^{-1}\mathbf{B}_1\right)^{-1}\left(\overline{\mathbf{A}}_2^{-1}\overline{\mathbf{A}}_1-\overline{\mathbf{B}}_2^{-1}\overline{\mathbf{B}}_1\right)\mathbf{A}_1^T\hat{\mathbf{p}}$$
$$\mathbf{q}_2=\frac{1}{2\pi i}\left(\overline{\mathbf{A}}_1^{-1}\mathbf{A}_2-\overline{\mathbf{B}}_1^{-1}\mathbf{B}_2\right)^{-1}\left(\overline{\mathbf{A}}_1^{-1}\mathbf{A}_1-\overline{\mathbf{B}}_1^{-1}\mathbf{B}_1\right)\mathbf{A}_1^T\hat{\mathbf{p}} \tag{3.36b}$$

Like those described in (3.20), (3.21) and (3.22), by canceling $\mathbf{f}_1(\zeta)$ and comparing the coefficients of corresponding terms, the unknown constants $\mathbf{c}_k$ can be determined to have the same expression as (3.28) except

$$\mathbf{t}_k=-\left(\mathbf{M}_1-\overline{\mathbf{M}}_2\right)\overline{\mathbf{A}}_2\overline{\mathbf{e}}_k-\left(\overline{\mathbf{M}}_1+\mathbf{M}_2\right)\mathbf{A}_2\mathbf{d}_k$$
$$=-\left(\mathbf{M}_1-\overline{\mathbf{M}}_2\right)\overline{\mathbf{A}}_2<\overline{\frac{-1}{k}\left(\frac{\gamma_\alpha}{\hat{\zeta}_\alpha}\right)^k}>\overline{\mathbf{q}}_2 \tag{3.37}$$

Having the solution of $\mathbf{c}_k$, substituting (3.33) and (3.36) into (3.20) can get function $\mathbf{f}_1(\zeta)$.

$$\mathbf{f}_1(\zeta)=\mathbf{A}_1^{-1}\mathbf{A}_2<\log\left(1-\frac{\gamma_\alpha}{\zeta_\alpha\hat{\zeta}_\alpha}\right)>\mathbf{q}_2+\sum_{k=1}^{\infty}\mathbf{A}_1^{-1}\left[\mathbf{A}_2<\gamma_\alpha^k>\mathbf{c}_k+\overline{\mathbf{A}}_2\overline{\mathbf{c}}_k\right]\zeta^{-k}$$

or

$$\mathbf{f}_1(\zeta)=\mathbf{B}_1^{-1}\mathbf{B}_2<\log\left(1-\frac{\gamma_\alpha}{\zeta_\alpha\hat{\zeta}_\alpha}\right)>\mathbf{q}_2+\sum_{k=1}^{\infty}\mathbf{B}_1^{-1}\left[\mathbf{B}_2<\gamma_\alpha^k>\mathbf{c}_k+\overline{\mathbf{B}}_2\overline{\mathbf{c}}_k\right]\zeta^{-k} \tag{3.38}$$

4. Discussion

In order to verify that the obtained solutions is true. This paper considers two simple examples to verify the solutions.

Case I:

The simplest situation pertains when the matrix and the inclusion are composed of the same material, such that $\mathbf{A}_1=\mathbf{A}_2=\mathbf{A}$, $\mathbf{B}_1=\mathbf{B}_2=\mathbf{B}$, $\mathbf{M}_1=\mathbf{M}_2=\mathbf{M}$ The solution herein is verified by dividing the proof into three parts.

1. Point Force or Dislocation on the matrix

From (3.15),(3.16) we can get

$$\mathbf{c}_k=\mathbf{G}_0^{-1}\mathbf{t}_k=\left[(\overline{\mathbf{M}}+\mathbf{M})\mathbf{A}\right]^{-1}\left(-i\mathbf{A}^{-T}\mathbf{e}_k\right)=\mathbf{e}_k$$
$$\mathbf{f}_1(\zeta)=-<\log\left(1-\frac{\gamma_\alpha}{\zeta_\alpha\hat{\zeta}_\alpha}\right)>\mathbf{q}_1-\sum_{k=1}^{\infty}\left[\mathbf{A}^{-1}\overline{\mathbf{A}}\overline{\mathbf{e}}_k-<\gamma_\alpha^k>\mathbf{c}_k-\mathbf{A}^{-1}\overline{\mathbf{A}}\overline{\mathbf{c}}_k\right]\zeta^{-k}$$
$$=-<\log\left(1-\frac{\gamma_\alpha}{\zeta_\alpha\hat{\zeta}_\alpha}\right)>\mathbf{q}_1+\sum_{k=1}^{\infty}<\gamma_\alpha^k>\mathbf{c}_k\,\zeta^{-k} \tag{4.1a}$$

where

$$\mathbf{q}_1=\begin{cases}\dfrac{1}{2\pi i}\mathbf{A}^T\hat{\mathbf{p}} \ , & \text{for point force}\\[2mm] \dfrac{1}{2\pi i}\mathbf{B}^T\hat{\mathbf{b}} \ , & \text{for dislocation}\end{cases} \tag{4.1b}$$

From (3.11),(3.13),(3.14) and (4.1) can get

$$
\begin{aligned}
\mathbf{f}_0(\zeta)+\mathbf{f}_1(\zeta) &= <\log(\zeta_\alpha-\hat{\zeta}_\alpha)>\mathbf{q}_1+\sum_{k=1}^{\infty}<\gamma_\alpha^k>\mathbf{c}_k\,\zeta^{-k} \\
&= \sum_{k=1}^{\infty}\mathbf{e}_k\,\zeta^k+\sum_{k=1}^{\infty}<\gamma_\alpha^k>\mathbf{c}_k\,\zeta^{-k} \\
&= \sum_{k=1}^{\infty}\mathbf{c}_k\,\zeta^k+\sum_{k=1}^{\infty}<\gamma_\alpha^k>\mathbf{c}_k\,\zeta^{-k} \\
&= \mathbf{f}_2^+(\zeta)+\mathbf{f}_2^-(\zeta)=\mathbf{f}_2(\zeta)
\end{aligned}
\tag{4.2}
$$

From (4.2), which are the same as are assumed for homogeneous materials

2. Point Force or Dislocation on the inclusion

Exchanging the condition for homogeneous materials for (3.28) yields

$$
\begin{aligned}
&\mathbf{G}_0=(\overline{\mathbf{M}}+\mathbf{M})\mathbf{A}\ ,\ \mathbf{G}_k=\mathbf{0}\ ,\ \mathbf{t}_k=-(\overline{\mathbf{M}}+\mathbf{M})\mathbf{A}\mathbf{d}_k \\
&\mathbf{c}_k=\mathbf{G}_0^{-1}\,\mathbf{t}_k=[(\overline{\mathbf{M}}+\mathbf{M})\mathbf{A}]^{-1}[-(\overline{\mathbf{M}}+\mathbf{M})\mathbf{A}\mathbf{d}_k]=-\mathbf{d}_k
\end{aligned}
\tag{4.3}
$$

From (3.17c), (3.24), (3.25) and (3.27) can get

$$
\begin{aligned}
&\mathbf{f}_0(\zeta)=<\log\zeta_\alpha>\mathbf{q}_1 \\
&\mathbf{f}_1(\zeta)=<\log\left(1-\frac{\gamma_\alpha}{\zeta_\alpha\hat{\zeta}_\alpha}\right)+\log\left(1-\frac{\hat{\zeta}_\alpha}{\zeta_\alpha}\right)+\log c_\alpha>\mathbf{q}_1 \\
&\mathbf{f}_0^*(\zeta)=<\log(\zeta_\alpha-\hat{\zeta}_\alpha)+\log\left(1-\frac{\gamma_\alpha}{\zeta_\alpha\hat{\zeta}_\alpha}\right)+\log c_\alpha>\mathbf{q}_1 \\
&\mathbf{f}_2(\zeta)=\sum_{k=1}^{\infty}\mathbf{c}_k\zeta^k+\sum_{k=1}^{\infty}<\gamma_\alpha^k>\mathbf{c}_k\zeta^{-k}=\sum_{k=1}^{\infty}-(\mathbf{d}_k\zeta^k+<\gamma_\alpha^k>\mathbf{d}_k\zeta^{-k})=0 \\
&\mathbf{d}_k=0
\end{aligned}
\tag{4.4}
$$

Where

$$
\mathbf{q}_1=\begin{cases}\dfrac{1}{2\pi i}\mathbf{A}^T\hat{\mathbf{p}} & ,\ \text{for point force} \\ \dfrac{1}{2\pi i}\mathbf{B}^T\hat{\mathbf{b}} & ,\ \text{for dislocation}\end{cases}
\tag{4.1b}
$$

Applying (4.4) yields the matrix function is the same as the inclusion function.

$$
\begin{aligned}
\mathbf{f}_0(\zeta)+\mathbf{f}_1(\zeta)&=\mathbf{f}_0^*(\zeta)+\mathbf{f}_2(\zeta) \\
&=<\log(\zeta_\alpha-\hat{\zeta}_\alpha)+\log\left(1-\frac{\gamma_\alpha}{\zeta_\alpha\hat{\zeta}_\alpha}\right)+\log c_\alpha>\mathbf{q}_1
\end{aligned}
\tag{4.5}
$$

3. Point Force or Dislocation on the interface of matrix and inclusion

(3.28) and (3.36) yield

$$
\begin{aligned}
&\mathbf{G}_0=(\overline{\mathbf{M}}+\mathbf{M})\mathbf{A}\ ,\ \mathbf{G}_k=\mathbf{0}\ ,\ \mathbf{t}_k=-(\overline{\mathbf{M}}+\mathbf{M})\mathbf{A}\mathbf{d}_k \\
&\mathbf{c}_k=\mathbf{G}_0^{-1}\,\mathbf{t}_k=[(\overline{\mathbf{M}}+\mathbf{M})\mathbf{A}]^{-1}[-(\overline{\mathbf{M}}+\mathbf{M})\mathbf{A}\mathbf{d}_k]=-\mathbf{d}_k \\
&\mathbf{q}_1=\mathbf{q}_2=\frac{1}{2\pi i}\mathbf{A}^T\hat{\mathbf{b}}\ ,\ \mathbf{q}_1'=\mathbf{0}\qquad \text{for point force} \\
&\mathbf{q}_1=\mathbf{q}_2=\frac{1}{2\pi i}\mathbf{B}^T\hat{\mathbf{b}}\ ,\ \mathbf{q}_1'=\mathbf{0}\qquad \text{for dislocation}
\end{aligned}
\tag{4.6}
$$

Using (3.32) ,(3.33) and (3.38), we have

$$
\begin{aligned}
&\mathbf{f}_0(\zeta)=<\log(\zeta_\alpha-\hat{\zeta}_\alpha)>\mathbf{q}_1 \\
&\mathbf{f}_1(\zeta)=<\log\left(1-\frac{\gamma_\alpha}{\zeta_\alpha\hat{\zeta}_\alpha}\right)>\mathbf{q}_2+\sum_{k=1}^{\infty}\left[<\gamma_\alpha^k>\mathbf{c}_k+\mathbf{A}_1^{-1}\overline{\mathbf{A}}_2\overline{\mathbf{c}}_k\right]\zeta^{-k} \\
&\mathbf{f}_0^*(\zeta)=<\log(\zeta_\alpha-\hat{\zeta}_\alpha)+\log\left(1-\frac{\gamma_\alpha}{\zeta_\alpha\hat{\zeta}_\alpha}\right)+\log c_\alpha>\mathbf{q}_2 \\
&\mathbf{f}_2(\zeta)=\sum_{k=1}^{\infty}\mathbf{c}_k\zeta^k+\sum_{k=1}^{\infty}<\gamma_\alpha^k>\mathbf{c}_k\zeta^{-k}=\sum_{k=1}^{\infty}-(\mathbf{d}_k\zeta^k+<\gamma_\alpha^k>\mathbf{d}_k\zeta^{-k})=0 \\
&\mathbf{c}_k=\mathbf{d}_k=\mathbf{0}
\end{aligned}
\tag{4.7}
$$

Combining the results of (4.7), one may prove that

$$
\begin{aligned}
\mathbf{f}_0(\zeta)+\mathbf{f}_1(\zeta)&=\mathbf{f}_0^*(\zeta)+\mathbf{f}_2(\zeta) \\
&=<\log(\zeta_\alpha-\hat{\zeta}_\alpha)>\mathbf{q}_1+<\log\left(1-\frac{\gamma_\alpha}{\zeta_\alpha\hat{\zeta}_\alpha}\right)>\mathbf{q}_2
\end{aligned}
\tag{4.8}
$$

we can get the same to have the function value on matrix and inclusion.

The result herein is therefore correct for homogeneous materials. It is also appropriate for other materials, so the result herein has wide applicability.

Case II:

The second case, reduction of the inclusion to a hole, such that $\mathbf{A}_1=\mathbf{A}$, $\mathbf{B}_1=\mathbf{B}$, $\mathbf{M}_1=\mathbf{M}$, $\mathbf{A}_2=\mathbf{B}_2=\mathbf{M}_2=\mathbf{0}$ is considered. Again, the proof is divided into three parts.

1. Point Force or Dislocation on the matrix

From (3.15) we can get

$\mathbf{G}_0 = \mathbf{0}$, $\mathbf{G}_k = \mathbf{0}$, $\mathbf{c}_k = \mathbf{0}$, $\mathbf{t}_k = -i\mathbf{A}_1^{-T}\mathbf{e}_k$ (4.9)

From (3.8)

$$\mathbf{f}_1(\zeta) = -\mathbf{f}_0^+(\zeta) - \mathbf{A}^{-1}\overline{\mathbf{A}}\overline{\mathbf{f}_0^-\left(\frac{1}{\zeta}\right)} \quad \text{for point force}$$

$$\mathbf{f}_1(\zeta) = -\mathbf{f}_0^+(\zeta) - \mathbf{B}^{-1}\overline{\mathbf{B}}\overline{\mathbf{f}_0^-\left(\frac{1}{\zeta}\right)} \quad \text{for dislocation} \tag{4.10}$$

Subtituting (3.10) into (4.10) can get

$$\mathbf{f}_1(\zeta) = -<\log\left(1-\frac{\gamma_\alpha}{\zeta_\alpha\hat{\zeta}_\alpha}\right)>\mathbf{q}_1 - \mathbf{A}^{-1}\overline{\mathbf{A}}<\log\left(\zeta_\alpha^{-1}-\bar{\hat{\zeta}}_\alpha\right)>\overline{\mathbf{q}}_1 \tag{4.11a}$$

or

$$\mathbf{f}_1(\zeta) = -<\log\left(1-\frac{\gamma_\alpha}{\zeta_\alpha\hat{\zeta}_\alpha}\right)>\mathbf{q}_1 - \mathbf{B}^{-1}\overline{\mathbf{B}}<\log\left(\zeta_\alpha^{-1}-\bar{\hat{\zeta}}_\alpha\right)>\overline{\mathbf{q}}_1 \tag{4.11b}$$

The result thus obtained is the same as was obtained by (Hwu 2004)

2. Point Force or Dislocation on the inclusion

From (3.28), can get

$\mathbf{G}_0 = \mathbf{0}$, $\mathbf{G}_k = \mathbf{0}$, $\mathbf{t}_k = \mathbf{0}$, $\mathbf{c}_k = \mathbf{0}$, $\mathbf{q}_1 = \mathbf{0}$ 4.12)

Using (3.20),(3.25),(3.26) may result in

$$\mathbf{f}_0(\zeta) = <\log\zeta_\alpha> \mathrm{d}$$
$$\mathbf{f}_1(\zeta) = \mathbf{f}_0^*(\zeta) = \mathbf{f}_2(\zeta) = \mathbf{0} \tag{4.13}$$

The result is the same as that the plates containing a hole

3. Point Force or Dislocation on the interface of matrix and inclusion

(3.28) and (3.36) yield

$\mathbf{G}_0 = \mathbf{G}_k = \mathbf{0}$, $\mathbf{t}_k = \mathbf{0}$, $\mathbf{c}_k = \mathbf{0}$, $\mathbf{q}_2 = \mathbf{q}_1' = \mathbf{0}$

(4.14)

$$\mathbf{q}_1 = \begin{cases} \dfrac{1}{2\pi i}\mathbf{A}^T\hat{\mathbf{p}} & \text{for point force} \\ \dfrac{1}{2\pi i}\mathbf{B}^T\hat{\mathbf{b}} & \text{for dislocation} \end{cases} \tag{4.15}$$

Using (3.32) ,(3.33) and (3.38)

$$\mathbf{f}_0(\zeta) = <\log\left(\zeta_\alpha - \hat{\zeta}_\alpha\right)> \mathbf{q}_1$$
$$\mathbf{f}_1(\zeta) = \mathbf{f}_0^*(\zeta) = \mathbf{f}_2(\zeta) = \mathbf{0} \tag{4.16}$$

The result thus obtained is similar to the plates containing a hole

The verification yields the results in earlier papers. Therefore, the result derived herein has been proven to be correct.

5. Conclusions

In this paper, the anisotropic elastic materials of stretching and bending problems were considered. General solutions for dislocation or point forces inside, outside, or on the interface between an anisotropic elliptical inclusion and matrix, were obtained. The solution is obtained more conveniently and quickly than other solutions and the applicable range is more extensive. Simple solutions are obtained by combining numerical solution to the singular integral equation. The presented analytical solutions can be applied to solve the problem of a crack penetrating an inclusion or lying around the interface, as well as the problem the interaction between a crack and an inclusion.

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