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Effect of Radiation on Natural Convection over an Isothermal Vertical Permeable Flat Plate in Porous Media

飽和多孔性介質內熱輻射效應對等溫垂直可穿透性平板自然對流熱傳遞之影響

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Abstract

In this paper laminar boundary layer solutions are numerically presented for the effect of radiation on free convection about an isothermal vertical permeable flat plate embedded in a saturated porous medium. These partial differential equations are transformed into the nonsimilar equations and then are solved by an implicit finite-difference method (Keller box method). Numerical results for the dimensionless temperature profiles and the local Nusselt number are presented for the blowing/suction parameter ξ, the radiation-conduction parameter R_d, and the surface temperature parameter H. The local Nusselt number increases (decreases) for the effect of suction (blowing). As the radiation-conduction parameter R_d and the surface temperature parameter H increase, the local Nusselt number also increases.

Key Words: Radiation, free convection, porous media

摘要

在本文中，吾人使用數值解法以探討在飽和多孔性介質內熱輻射效應對等溫垂直可穿透平板自然對流熱傳遞之影響。偏微分方程式經過變數轉換後，形成一組非相似邊界層方程式，再以一隱性有限差分方法（凱勒盒子法）解之。數值計算結果主要顯示噴吸流參數ξ、輻射－傳導參數R_d與表面溫度參數 H，對無因次溫度分佈與局部 Nusselt 數之影響。局部 Nusselt 數因吸（噴）流效應而增大（減小）。此外，增加輻射－傳導參數R_d與增加表面溫度參數 H，則會增強局部 Nusselt 數。

關鍵字：輻射，自然對流，多孔性介質

1. Introduction

The convective heat transfer in a saturated porous medium has a number of important applications in geothermal and geophysical engineering. These include nuclear reactor cooling system, extraction of geothermal energy, thermal insulation of buildings, filtration processes and disposal of underground nuclear wastes.

In the aspect of free convection with impermeable wall, natural convection about a vertical flat plate embedded in a porous medium with application to heat transfer from a dike was studied by Cheng and Minkowycz [1] by similarity solution. Na and Pop [2] presented the free convection flow past a non-isothermal vertical flat plate embedded in a saturated porous medium using the Box-method. Gorla and Zinalabedini [3] investigated free convection from a vertical plate with nonuniform surface temperature and embedded in a saturated porous medium. Free convection in a saturated porous medium adjacent to a non-isothermal vertical impermeable wall was presented by Seetharamu and Dutta [4]. Nakayama and Hossain [5] studied the free convection in a saturated porous medium beyond the similarity solution.

In the aspect of free convection with permeable wall, the influence of lateral mass flux on free convection boundary layers in a saturated porous medium was studied by Cheng [6]. The similarity solution is possible only when the variations of the wall temperature and the transpiration rate are proportional to power-law of x measured from the leading edge. Magyari and Keller [7] presented the exact analytical solutions for free convection boundary layers on a heated vertical plate with lateral mass flux embedded in a saturated porous medium. From the practical point of view, however, the uniform mass flux may be easily realized. Merkin [8], Minkowycz and Cheng [9] investigated the effect of uniform surface mass flux on a vertical flat plate with uniform wall temperature by series method and local non-similarity solution, respectively. Bakier et al. [10] studied the nonsimilar solutions for free convection from a vertical plate in porous media. Previous researches [1-10], however, have only concentrated upon the problem with no radiation effect.

As the difference between the surface temperature and the ambient temperature is large, it may cause the radiation effect to become important. The problem of radiation effect on convection flow has many important applications such as space technology and processes involving high temperatures such as the geothermal engineering, the sensible heat storage bed, the nuclear reactor cooling system and the underground nuclear wastes disposal.

In the aspect of convection-radiation in porous media, Hossain and Pop [11] first studied the radiation effect on free convection flow along an inclined surface placed in porous media. Then, Raptis [12] investigated the radiation and free convection flow through a porous medium bounded by a vertical infinite porous plate. Yih [13] studied the radiation effect on free convection over an impermeable vertical cylinder embedded in porous media. Thermal radiation effects on power-law fluid over a horizontal plate embedded in a porous medium was presented by Mohammadein and El-Amin [14]. Hossain and Pop [15] investigated the radiation effects on free convection over a vertical flat plate embedded in a porous medium with high porosity. In above researches [11-15], the Rosseland diffusion approximation is employed. This approximation leads to a considerable simplification in the expression for radiant flux.

In the present analysis, we extend the previous work of Minkowycz and Cheng [9] to investigate the effect of radiation on the heat transfer characteristics in free convection over an isothermal vertical permeable flat plate embedded in porous media by using the implicit finite difference method together with Keller box elimination technique.

2. Analysis

Consider the problem of the radiation effect on free convection boundary-layer flow of optically dense viscous incompressible fluids over an isothermal vertical permeable flat plate embedded in a saturated porous medium. The wall temperature of the vertical flat plate T_w is higher than the ambient temperature T_∞. The x coordinate is measured along the flat plate from the leading edge and the y coordinate is measured normal to the flat plate. The gravitational acceleration g is acting downward along the flat plate, opposite to the x-direction. The surface blowing/suction velocity V_w is also uniform. The variations of fluid properties are limited to density variation that affects the buoyancy force term only. The viscous dissipation effect is neglected for the low velocity. The radiative heat flux in the x-direction is negligible in comparison with that in y-direction.

The governing equations for the problem under consideration with the boundary layer, Boussinesq and Rosseland diffusion approximations and the Darcy law can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad (1)$$

$$\frac{\partial u}{\partial y} = \frac{g\beta K}{\nu}\frac{\partial T}{\partial y}, \qquad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma}{3(a_r+\sigma_s)\rho C_p}\frac{\partial}{\partial y}\left(T^3\frac{\partial T}{\partial y}\right). \qquad (3)$$

The boundary conditions are defined as follows:

$$y = 0: v = V_w, \qquad T = T_w, \qquad (4)$$

$$y \to \infty: u = 0, \qquad T = T_\infty, \qquad (5)$$

where u and v are the Darcian velocities in the x and y directions, respectively. β is the thermal expansion coefficient of the fluid. K is the permeability of the porous medium. ν is the kinematic viscosity of the fluid. T is the temperature of the fluid and the porous medium which are in local thermal equilibrium. α is the equivalent thermal diffusivity. σ is the Stefan-Boltzmann constant. a_r is the Rosseland mean extinction coefficient. σ_s is the scattering coefficient. ρ is the density of the fluid. C_p is the specific heat at constant pressure. The term $16\sigma T^3/[3(a_r+\sigma_s)]$ can be considered as the "radiative conductivity".

We now define a stream function ψ such that

$$u = \partial\psi/\partial y \quad \text{and} \quad v = -\partial\psi/\partial x. \qquad (6)$$

The continuity Eq. (1) is then

automatically satisfied. Invoking the following dimensionless variables:

$$\xi = \frac{V_w x}{\alpha Ra_x^{1/2}} \quad (7a)$$

$$\eta = \frac{y Ra_x^{1/2}}{x} \quad (7b)$$

$$f(\xi,\eta) = \frac{\psi}{\alpha Ra_x^{1/2}} \quad (7c)$$

$$\theta(\xi,\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (7d)$$

$$Ra_x = \frac{g\beta(T_w - T_\infty)Kx}{\nu\alpha} \quad (7e)$$

and substituting Eq. (7) into Eqs. (1)-(5), we obtain the following transformed governing equations

$$f' = \theta, \quad (8)$$

$$\theta'' + \frac{1}{2}f\theta' + \frac{4R_d}{3}\left\{\theta'\left[(H-1)\theta+1\right]^3\right\}' = \frac{\xi}{2}\left(f'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial f}{\partial\xi}\right). \quad (9)$$

The boundary conditions are defined as follows:

$$\eta = 0: \quad f = -\xi, \quad \theta = 1, \quad (10)$$

$$\eta \to \infty: \quad \theta = 0. \quad (11)$$

In the above equations, the primes denote the differentiation with respect to η. Equation (8) is obtained by integrating Eq. (2) once with the help of Eq. (5). Ra_x is the modified local Rayleigh number for the flow through the porous medium. ξ is the blowing/suction parameter. For the case of blowing, $V_w > 0$ and hence $\xi > 0$. On the other hand, for the case of suction, $V_w < 0$ and hence $\xi < 0$. $R_d = 4\sigma T_\infty^3/[k(a_r + \sigma_s)]$ is the radiation-conduction parameter. $H = T_w/T_\infty$ is the surface temperature parameter.

In terms of the new variables, the velocity components are given by

$$u = \frac{\alpha Ra_x}{x} f', \quad (12)$$

and

$$v = -\frac{\alpha Ra_x^{1/2}}{2x}\left(f + \xi\frac{\partial f}{\partial\xi} - \eta f'\right). \quad (13)$$

The heat flux q_w at the surface of the vertical plate is

$$q_w = -\left\{\left[k + \frac{16\sigma T^3}{3(a_r + \sigma_s)}\right]\frac{\partial T}{\partial y}\right\}_{y=0}. \quad (14)$$

For practical applications, it is usually the local Nusselt number that is of interest. This can be expressed as

$$Nu_x = \frac{hx}{k} = \frac{q_w x}{k(T_w - T_\infty)} \quad (15)$$

where h denotes the local heat transfer coefficient and k represents the thermal conductivity. Substituting Eqs. (7b) and (14) into Eq. (15), we obtain

$$\frac{Nu_x}{Ra_x^{1/2}} = -\left(1 + \frac{4R_d H^3}{3}\right)\theta'(\xi,0). \quad (16)$$

It may be noticed that for the case of $R_d = 0$, Eqs. (8)-(9) are reduced to those of Minkowycz and Cheng [9] where a nonsimilar solution was obtained previously.

3. Numerical method

The above governing Eqs. (8)-(9) and the boundary conditions (10)-(11) are nonlinear partial differential equations depending on the blowing/suction parameter ξ, the radiation-conduction parameter R_d, and the surface temperature parameter H. The present analysis integrates the system of Eqs. (8)-(9) by the implicit finite difference approximation together with the modified Keller box method of Cebeci and Bradshaw [16]. The computations were carried out on a personal computer with $\Delta\xi$ = 0.05

(uniform grid), the first step size $\Delta\eta_1$ = 0.01, the variable grid parameter is chosen 1.01. The value of η_∞ = 100 was found to be sufficiently accurate for $|\theta_\infty| < 10^{-3}$. The requirement that the variation of the temperature distribution is less than 10^{-5} between any two successive iterations is employed as the criterion of convergence.

4. Results and discussion

In order to verify the accuracy of the present method, we have compared our results with those of Minkowycz and Cheng [9]. Table 1 shows the values of $-\theta'(\xi,0)$ for various values of ξ with R_d = 0. The comparison in the above case is found to be in good agreement.

Table 1. Comparison of values of $-\theta'(\xi,0)$ for various values of ξ with R_d = 0.

ξ	Minkowycz and Cheng [9]	Present results
−2	2.002	2.0015
−1	1.068	1.0725
0	0.4438	0.4437
1	0.1423	0.1416
2	0.0335	0.0333

Numerical results are presented graphically for the blowing/suction parameter ξ ranging from −2 to 2, the radiation-conduction parameter R_d ranging from 0 to 10, and the surface temperature parameter H ranging from 1.1 to 3.0.

Figure 1 shows the dimensionless temperature profiles for various values of ξ with R_d = 10, H = 1.1. The effect of suction decreases the thermal boundary layer. Whereas, for the case of blowing increases the thermal boundary layer.

Figure 2 illustrates the dimensionless temperature profiles for various values of H with $\xi = -1$, R_d = 1. It is observed that the dimensionless temperature profiles increase due to the increase in the surface temperature parameter H. As the value of H increases radiation absorption in the boundary layer increases, causing the dimensionless temperature profiles become large

The dimensionless temperature profiles for various values of R_d with ξ = 2, H = 1.5 is displayed in Fig. 3. Increasing the radiation-conduction parameter R_d enhances the dimensionless temperature profiles. This is because in the presence of radiation the temperature is large. Moreover, when R_d and H are changed, the dimensionless temperature profiles tend to behave in a similar pattern (see Figs. 2-3). The dimensionless temperature profiles decrease monotonically from the surface to the ambient, as shown in Figs. 1-3.

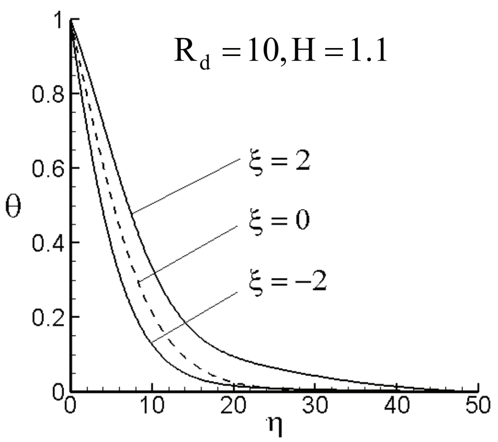

Fig. 1. Dimensionless temperature profiles for various values of ξ

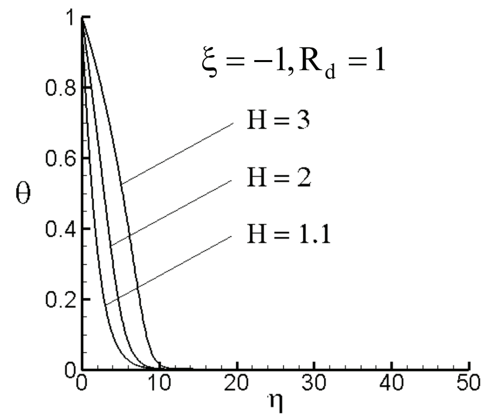

Fig. 2. Dimensionless temperature profiles for various values of H

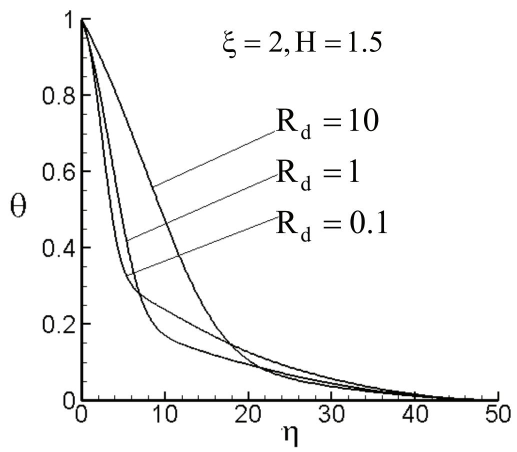

Fig. 3. Dimensionless temperature profiles for various values of R_d

Table 2 shows the local Nusselt number for various values of the blowing/suction parameter ξ with R_d = 10, H = 1.1. The local Nusselt number is increasing for the effect of suction, i.e. ξ < 0. The opposite is true for the effect of blowing. This is because that suction (blowing) case decreases (increases) the thermal boundary layer (Fig. 1). Table 3 illustrates the local Nusselt number for various values of the surface temperature parameter H with $\xi = -1$, R_d = 1. Increasing the surface temperature parameter H enhances the local Nusselt number. The local Nusselt number for various values of radiation-conduction parameter R_d with ξ = 2, H = 1.5 is shown in Table 4. It is seen that the value of the local Nusselt number increases with increasing the value of R_d. In the pure convection heat transfer, the local Nusselt number is only proportional to the dimensionless surface temperature gradient $-\theta'(\xi,0)$. For the case of large H and R_d (radiation effect becomes pronounced), although the $-\theta'(\xi,0)$ is low as shown in Figs. 2-3, the local Nusselt number is still large. This is because the local Nusselt number is found to be more sensitive to H and R_d than $-\theta'(\xi,0)$, as revealed in Eq. (16).

Table 2. Local Nusselt number for various values of ξ with R_d = 10, H = 1.1.

ξ	$Nu_x / Ra_x^{1/2}$
−2	2.9019
−1	2.3378
0	1.8549
1	1.4518
2	1.1143

Table 3. Local Nusselt number for various values of H with $\xi = -1$, R_d = 1.

H	$Nu_x / Ra_x^{1/2}$
3.0	2.5285
2.5	2.1103
2.0	1.7457
1.5	1.4501
1.1	1.2761

Table 4. Local Nusselt number for various values of R_d with ξ = 2, H = 1.5.

R_d	$Nu_x / Ra_x^{1/2}$
10	1.8692
5	1.1628
1	0.3341
0.5	0.1883
0.1	0.0627

5. Conclusions

The radiation effect on the free convection flow of an optically dense viscous fluid adjacent to an isothermal vertical permeable flat plate embedded in a saturated porous medium with Rosseland diffusion approximation is numerically investigated. The transformed nonsimilar conservation equations are obtained and solved by the Keller box method. Numerical results are given for the dimensionless temperature profiles and the local Nusselt number for various values of the blowing/suction parameter ξ, the radiation-conduction parameter R_d, and the surface temperature parameter H. It is apparent that decreasing the blowing/suction parameter ξ (for the case of suction) and increasing the radiation-conduction parameter R_d and the surface temperature parameter H will enhance the local Nusselt number.

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