

## 以離散渦流法探討雙平行噴流流向演進特性

### Studies on the Spatial Evolution of Vortex Parallel Jets by Discrete Vortex

#### Method

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#### 摘要

本文提出一渦流模式模擬研究雙平行噴流之流向演進特性，平均速度、紊流強度及雷諾應力被以長時間平均方式求算出，並將所得計算之結果與現存實驗值做一比較，發現二者俱極佳之相合性。在本文之計算中雙平行噴流發展之三個特徵區域可被明顯地區分出，經由對稱面速度發展之比較，可訂出整合點及結合點之位置，並發現紊流強度在結合點後逐漸趨於一定值，將此定值與雙平面噴流與單平面噴流此區域之值做比較，發現在定量上幾乎一致。另外，為研究結合點附近流體之周期性運動表現，不同噴口比( $S/W$ )之結合點附近的流向速度頻譜被分析，發現在  $5/3 < S/W < 2$  範圍內，速度頻譜圖俱明顯之主導頻率，此主導頻率化為無因次之簡約頻率，發現  $S/W$  越小，則簡約頻率也越小。

**關鍵字：**雙平行噴流、結合點、紊流特性、渦流模式

#### Abstract

The objective of this study was to investigate the spatial evolutions of statistical turbulent properties for parallel plane jets using the vortex method. The mean velocities, turbulent intensities, and Reynolds shear stress were calculated using long time average processes and were compared with experiments. In the comparisons of centerline velocity ( $U_c$ ) along the stream-wise direction, three divided regions, which are the converging region, the merging region, and the combined region, could be clearly identified and good agreement to experimental results was found. In the combined region, the two jets combine to form a single jet flow. Periodic behavior was observed in the near-field region between parallel jets as  $5/3 < S/W < 2$ . As the nozzle spacing decreased, the introduced frequency decreased. The phenomena could be attributed to buff body shedding in the near field and a confining effect of the outer shear layers.

**Key Words:** parallel plane jet, combined point, turbulence properties, vortex model

## 1. Introduction

Parallel turbulent plane jets have numerous technological applications such as in the entrainment and mixing process in boiler and gas turbine combustion chambers, injection, and waste disposal plumes from stacks. The flow field of two parallel plane jets is complex and contains rich vortical scales during the spatial evolution. The flow pattern for parallel plane jets, which are schematically shown in Fig. 1, is produced by a subsonic flow of air issuing from two identical plane parallel nozzles restricted by two plates, where the two plane nozzles, which are with width ( $W$ ), are separated in the transverse ( $Y$ ) direction by  $S$ . The dominant parameter for the present flow field is the jet spacing ( $S/W$ ).

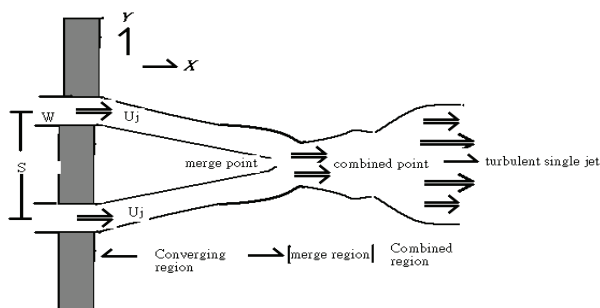


Fig. 1: Description of a parallel jet flow field.

Previous results indicate that the flow-field can be divided into three regions: (1) the converging region; (2) the merging region; and (3) the combined region. In the flow-field, a sub-atmospheric pressure zone close to the nozzle plate causes the individual jets to curve towards each other in a region and merge together at some downstream distance ( $X_{mp}$ ), known as the merging point, where the mean velocity ( $U_c$ ) on the symmetric axis is zero.

Downstream from the merging point (mp) in the merging region, the two jets continue to interact with each other up to the combined point (cp) where the mean stream-wise velocity on the symmetric axis ( $U_c$ ) attains its maximum value ( $U_{c_{max}}$ ). Downstream from the combined point, in the combined region, the two jets combine to form a single jet flow.

Single turbulent plane and offset jets, which are of great practical and theoretical importance, have been studied extensively [1, 2]. However, far fewer investigations into the behavior of parallel turbulent plane jets appear in the literature. The earliest studies were conducted by Tanaka [3, 4], and he also identified three definite regions of the flow-field in the stream-wise direction. In the experiments of Elbanna et al. [5], the assumption that the parallel jets would become a single jet after the merging point and attain a similar trend further downstream was confirmed. Lin and Sheu [6] studied and compared the spatial evolutions of parallel turbulent plane jets with and without an end wall between nozzles at the exit. Kanz et al. [7] studied the developments of a confined parallel jet and found that three dimensional effects produce rapid mixing caused by a strong secondary flow. In Nasr and Lai's experiment [8], the acoustic excitation introduced at the outer shear layer mode has been shown to reduce the size of the potential core, re-circulation zone, merging length, and combined length but enhance jet spreading, stream-wise velocity decay, and volume entrainment. Ko and Lau [9] found respective trains of coherent structures in the merging and combined regions. By changing the jet

spacing, Snyder and Christensen [10] found that periodic behavior in the near-field region between parallel jets as  $S/W < 2$ . Furthermore, as the nozzle spacing decreased, the introduced frequency decreased.

In numerical studies, Anderson and Spall [11] presented experimental and numerical results for plane parallel jets at different spacing; Spall [12] studied the influence of buoyancy on the spatial evolutions of parallel jets; and Spall et al. [13] found an integral constant along the stream-wise direction. The above results were obtained by solving the Reynolds-averaged Navier-Stokes equations employing a standard  $k-\varepsilon$  turbulent model. Apparently, the spatial evolutions of twinjets are dominated by the interactions of vortex structures at different sizes. The turbulent properties are anisotropic. To obtain the more reliable spatial evolution of parallel turbulent plane jets, a more satisfactory numerical process is needed.

The vortex method has been widely used to study various types of vortical flow, such as mixing layers, wake, jet, oscillating wing, and separated flow of blunt-bodies [14, 15, and 16]. Due to the success of vortex simulation on interactions of vortical structures in free shear layers, the vortex simulating technique is suitable for researching the evolution of a flow field, which is dominated by vortical structures. In the present paper, a vortex model is introduced to study the spatial behaviors of parallel turbulent plane jets. In order to confirm the reliability of the present vortex model, the mean velocities, mean flow directions,

turbulent intensities, and Reynolds shear stress at various stream-wise locations is calculated using long time mean processes and were compared with experiments.

## 2. Vortex Model

For parallel plane jets, flow issues from two identical plane parallel nozzles restricted by two plates. The geometry parameters and flow field are shown in Fig. 1. The flow field can be divided into three elements: (1) the solid wall around the exit, (2) the flow source at the exit, and (3) the vortex release from the edge of the exits. Because Nasr and Lai [8] have found that the changing of Reynolds number makes little effect on the flow properties, the inviscid vortex model is proposed to study the spatial development of the present flow-field.

In the present vortex model, the effects of solid walls and flow sources on the exits can be thought of as a potential problem, so they can be modeled by the proper selection of singularity distribution. The edges of the jet exit are the sources of the free vortices, and can be viewed as a separated boundary, so the circulation production rate can be modeled as:

$$\frac{d\Gamma}{dt} = \int_0^\delta \omega U dy \approx \int_0^\delta -\frac{\partial U}{\partial y} U dy = +, -\frac{1}{2} U_j^2, \quad ,$$

where  $\delta$  is the boundary layer thickness at the edge of the jet exits and  $U_e$  is the jet exit velocity. The strength of a new point vortex shed during a short time interval can be approximated by

$$\Delta\Gamma \approx +, -\frac{1}{2} U_j^2 \Delta t.$$

The unsteady velocity field is

$$\vec{U}(x, y, t) = \vec{U}_{wall} + \vec{U}_{jet} + \vec{U}_{free \text{ vortices}},$$

where  $\vec{U}_{wall}$ ,  $\vec{U}_{jet}$ , and  $\vec{U}_{free \text{ vortices}}$  are the velocities introduced by a solid wall, jet exits and free vortices, respectively. At any instant, the boundary conditions  $\vec{U} \cdot \vec{n} = 1$  at the jet exits and  $\vec{U} \cdot \vec{n} = 0$  at the solid wall must be met. Therefore, the strengths of the sources at the solid wall and jet exits must be solved and the problem can be reduced as a system of linear equations expressed in the form:

$$A \cdot \vec{S} + \text{normal velocities by free vortices} = 0$$

, where  $A$  is the geometric matrix and  $\vec{S}$  is the source strength vector of the solid wall and jet exits. Because the geometric matrix is invariant to time, LU decomposition is adopted to solve linear equations for enhancing computational efficiency.

### 3. Discussion

The case at  $S/W=4$  is adopted to study the flow field in the following paragraphs. The mean stream-wise velocity field for  $S/W=4$  is presented in Fig. 2. This figure illustrates that the flow-field has both jet-like and wake-like characteristics in the near field. Studying only the inner two shear layers, the velocity distribution in the transverse direction resembles the wake behind a bluff body with dimension  $W$ . However, studying only the outer two shear layers, the velocity field behaves as a typical plane jet. In the far field, the merged jets have the mean flow characteristics of a single plane jet. The mean transverse velocity field is shown in Fig. 3. In this figure, one can see the upper plane jet deflected in the downward

direction and the lower plane jet deflected in the upward direction. The two jets merge together at about  $X/S = 1.4$ .

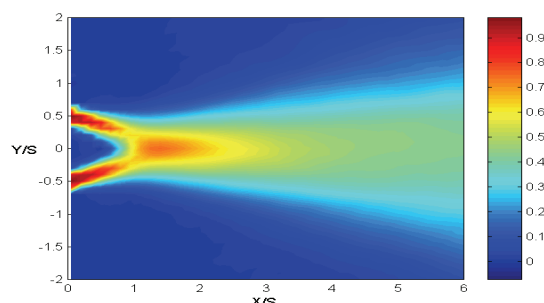


Fig.2: The mean stream-wise velocity field.

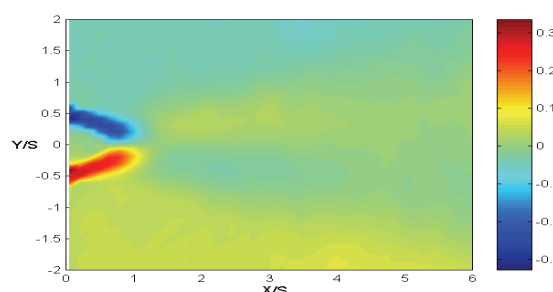


Fig. 3: The mean transverse velocity field in the near field.

The stream-wise and transverse turbulence intensity fields,  $u'$  and  $v'$  are indicated in Fig. 4(a) and Fig. 4 (b), respectively. In Fig. 4(a), it was found that the stream-wise turbulence intensity appears large at about  $X/S=1$  and then becomes smaller downstream. In Fig. 4(a), the two plane jet combination can be clearly observed. In the transverse turbulence intensity field (Fig. 4(b)), a high intensity spots can be seen near  $X/S=1$ . Comparing Fig. 3, Fig. 4(a) and Fig. 4(b), as the two plane jets combination appears, high turbulence intensity regions can be seen. The high intensity stream-wise turbulence intensity region appears and follows a low intensity area. When the stream-wise turbulence

intensity becomes lower, the high transverse turbulence intensity region stands in the flow field, meaning that in the processes of the two plane jets combining, the stream-wise turbulence is generated firstly as two inner shear layers contacting together and then the flapping vortices are formed and the high transverse turbulence intensity is produced. The Reynolds field is indicated in Fig. 4(c). A high Reynolds stress area is observed near  $X/S=1$  and this area spreads from the two plane jet exits to the two inner shear layer contacting point. Therefore, high turbulence production can be approached in the jet flow field by letting multi-jet contact appear.

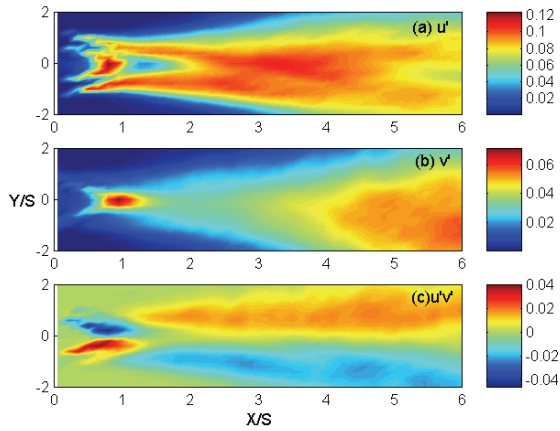


Fig. 4: (a) The stream-wise turbulence intensity field, (b) transverse turbulence intensity field root, (c) Reynolds stress field.

The spatial development of the mean x-component velocity is plotted in Fig. 5. Deflected velocity distribution can be seen in the symmetric axis up to  $X/S=1.4$ . Downstream of the combined point, at about  $X/S=2$ , the two jets combine to spread like a single free jet.

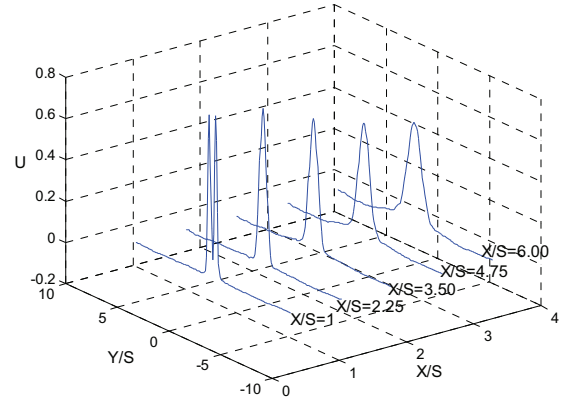


Fig. 5: The mean stream-wise velocity distribution at various stream-wise locations.

Figs. 6 and 7 display the spatial development of stream-wise and transverse turbulence intensities, respectively. The distributions of the stream-wise turbulence intensity,  $u'$ , exhibit two peaks in the inner and outer shear layers upstream of the merging point. The  $u'$  peak has a higher magnitude in the outer shear layer than that in the inner shear layer in the converging region. This phenomenon indicates that  $u'$  in the outer shear layer is less constrained by the converging process. After  $X/S=3$ , both the distributions of  $u'$  and  $v'$  exhibit only one peak in the symmetric axis. Therefore, the two plane jets merge together completely after  $X/S=3$  and the spatial evolution of the flow-field downstream is dominated by a single turbulent plane jet.

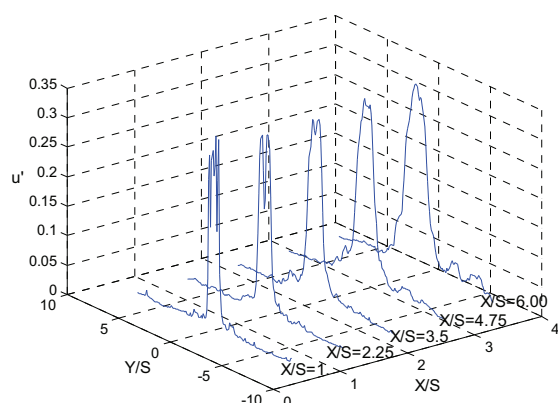


Fig. 6: The stream-wise fluctuant velocity distribution at various stream-wise locations.

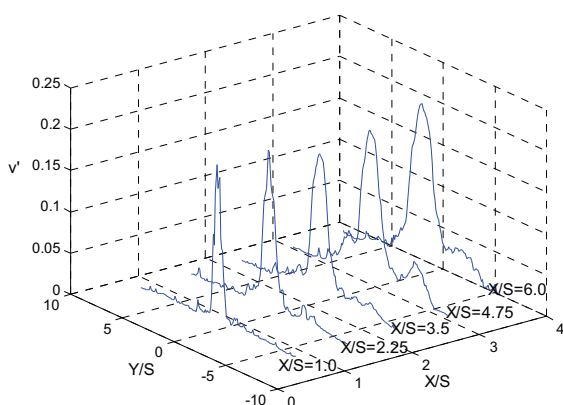


Fig. 7: The transverse fluctuant velocity distribution at various stream-wise locations.

The spatial development of Reynolds stress is presented in Fig. 8. In Figs. 4 and 8, it can be seen that Reynolds shear stresses in the inner and outer shear layers are similar magnitude close to the nozzle exit. A local maximum in the Reynolds shear stress appears near  $X/S=1$ , just upstream of the merging point. This means that high momentum transportation appears near the merging point between the two plane jets. A high rate of turbulent production is achieved; therefore, the high stream-wise and transverse turbulence

intensity regions follow.

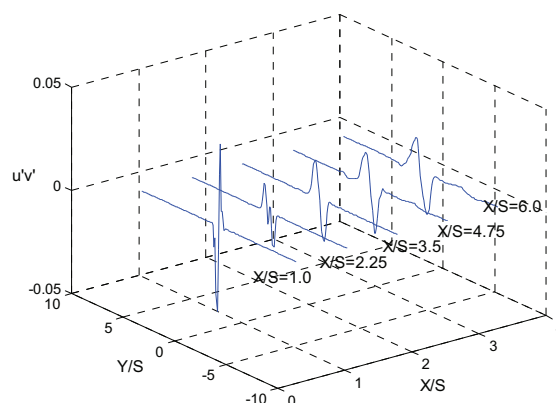


Fig. 8: Reynolds stress distribution at various stream-wise locations.

Due to the mutual entrainment of the two jets, a sub-atmospheric region between the nozzle plate and inner shear layers is formed, resulting in the individual jet's axis being deflected towards the axis of symmetry. Generally, the flow field of the two parallel jets can be divided into three distinct regions: the converging region, the merging region, and the combined region. The converging region includes a recirculation zone which is bounded by the common wall of the two nozzles and the inner layers of the individual jets. The merge point (mp) is the position of the termination of the recirculation zone. The combined point (cp) is the place where the X-component velocity on the axis of symmetry attains maximum. The variation of the mean stream-wise velocity on the axis of symmetry with a downstream direction is indicated in Fig. 9. It can be seen that  $\overline{U_c}$  is almost zero in the converging region, thus indicating dead flow. At the merging region,  $\overline{U_c}$  continues to increase up to the combined point due to

the merging process. Downstream of the combined point, the two individual jets resemble a single jet and  $\overline{U_c}$  decreases as the jet spreads.

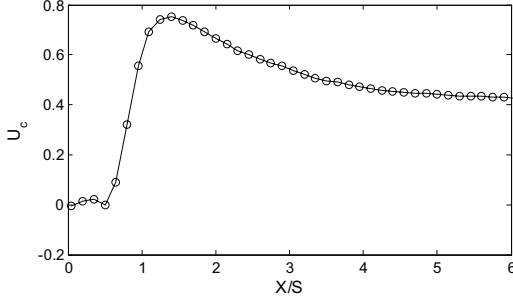


Fig. 9: The mean stream-wise velocity of the symmetric plane at various locations.

Comparison of the merge and combined point results from the present investigation and several other previous investigations are shown in Table 1. The comparison shows good consistency between the present results and those of previous experiments. The merge point is the point of zero velocity on the symmetry plane and is easily determined from the present calculations. Experimentally, this point is more difficult to determine due to hot-wire sensor's lack of reverse flow sensitivity. From Table 1, it can be seen that as the jet spacing ( $S/W$ ) increases, the position of the combined point ( $X_{cp}/S$ ) approaches 1, and as the jet spacing decreases,  $X_{cp}/S$  increases. The same phenomenon has also been observed in experiments [11].

Table 1: The comparison of merge points and combined points

| S/W      | 2 <sup>*0</sup> | 4 <sup>*0</sup> | 4.25 <sup>*1</sup> | 6 <sup>*2</sup> | 30 <sup>*3</sup> |
|----------|-----------------|-----------------|--------------------|-----------------|------------------|
| $X_{mp}$ | 0.5             | 0.5             | 0.94               | 1.1             | 0.72             |
| $X_{cp}$ | 2.5             | 1.5             | 1.87               | 2.1             | 1.1              |

<sup>\*0</sup>: Present results

<sup>\*1</sup>: Nasr and Lai's Experimental results [8]

<sup>\*2</sup>: Miller's Experimental results [17]

<sup>\*3</sup>: Lin and Sheu's experimental results [6]

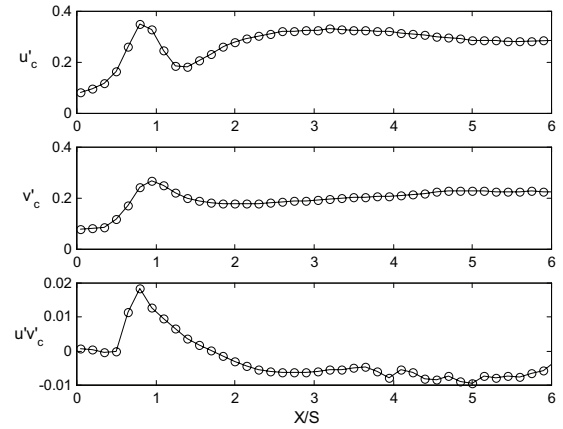


Fig. 10: (a) The spatial evolution of stream-wise turbulence intensity of the symmetry plane, (b) spatial evolution of transverse turbulence intensity of the symmetry plane, (c) spatial evolution of Reynolds stress of the symmetry plane.

The turbulence intensities of stream-wise and transverse velocities along the symmetric plane are shown in Figs. 10(a) and (b). The stream-wise location of rapid increase in  $u'_c$  is very close to the merge point. As  $X/S$  exceeds 5,  $u'_c$  and  $v'_c$  approach constant values, respectively. The same phenomena were observed in experiments [11]. Table 2 shows values of  $u'_c$  and  $v'_c$  at far-downstream for the present results and the last experiments. In Table 2, the values of  $u'_c$  and  $v'_c$  at far-

downstream are almost the same for different  $S/W$ . The values from parallel plane jets are equal to those from a similar region of a single turbulent plane jet by Gutmark and Wygnanski [2]. The results indicate that at far-downstream, the two jets resemble a single jet, which is with similar properties.

Table 2: The comparison of  $u'_c$  and  $v'_c$  at far-downstream.

|        | Present results<br>( $S/W=4$ ) | Anderson and Spall's [11]<br>( $S/W=9$ ) | Anderson and Spall's [11]<br>( $S/W=13$ ) | Gutmark and Wygnanski's [2]<br>(single jet)* |
|--------|--------------------------------|------------------------------------------|-------------------------------------------|----------------------------------------------|
| $u'_c$ | 0.27                           | 0.25                                     | 0.25                                      | 0.25                                         |
| $v'_c$ | 0.21                           | 0.2                                      | 0.24                                      | 0.2                                          |

\* Gutmark and Wygnanski's results are from a similar region of the turbulent plane jet.

The dynamic behavior of the free shear layer is governed by the formation and evolution of the flow instabilities. Nasr and Lai [8] showed that acoustic excitation at the outer layer mode reduced the potential core, recirculation zone, merging length, and combined length but enhanced jet spreading, stream-wise velocity decay, and volume entrainment. Snyder and Christensen [10] found periodic behavior in the near-field region between parallel jets in some range of jet spacing ( $S/W$ ). The spectrums of stream-wise fluctuant velocities at the neighbors of combined points for  $S/W=4$ , 2,  $5/3$  and  $4/3$  are displayed in Fig. 11.

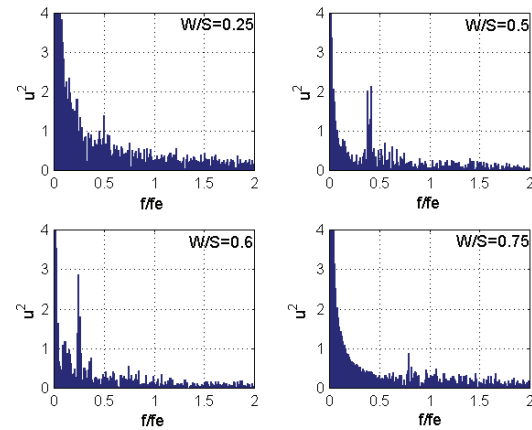


Fig. 11: The spectrums of stream-wise fluctuant velocities at the neighbors of combined points for various  $S/W$ .

Clear dominant frequencies are observed in the cases for  $S/W=2$  and  $5/3$ . Therefore, the periodic behavior in the near-field region between parallel jets appears only in some range of  $S/W$ . In Fig. 11, as  $W/S$  increases, the dominant frequency decreases. A comparison of dominant reduced frequencies ( $\frac{fd}{U_j}$ ) at

various  $S/W$  is indicated in Table 3. In order to more clearly understand the behavior of periodic motion, the instantaneous fluctuant stream-wise velocity fields for  $S/W=2$  and 4 are shown in Fig. 12. In the case  $S/W=4$ , no clear dominant frequency is found in the spectrum and no clear vortex structures are found neighboring the merge point. However, not only does a clear dominant frequency appear in the spectrum, but apparent vortical structures exists neighboring the merge point for  $S/W=2$ .

Table3: A comparison for dominant reduced frequencies ( $\frac{fd}{U_j}$ ) at various S/W.

|                  | present <sup>*1</sup> | present <sup>*2</sup> | Exp. <sup>*3</sup> | Exp. <sup>*4</sup> |
|------------------|-----------------------|-----------------------|--------------------|--------------------|
| S/W              | 2                     | 5/3                   | 7/6                | 3/2                |
| $\frac{fd}{U_j}$ | 0.39                  | 0.26                  | 0.333              | 0.24               |

<sup>\*1,\*2</sup>: Present results

<sup>\*3,\*4</sup>: Experimental results from reference 10.

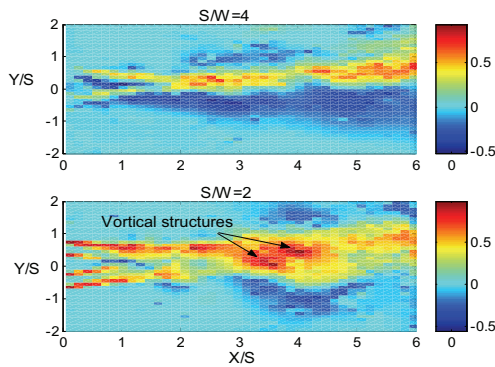


Fig. 12: (a) Instantaneous fluctuant stream-wise velocity field for S/W=4 (b) instantaneous fluctuant stream-wise velocity field for S/W=2.

The increase in reduced frequency with increasing S/W is due to the confining effect of the outer shear layers. As S/W increases, the outer shear layers are brought closer to the bluff body shedding region between the jets. This had an effect similar to that of confining walls in a wind tunnel being brought closer to a bluff body. The walls of the wind-tunnel alter the trajectory of the streamline from that which could exist in an unconfined configuration resulting in an increase in the shedding frequency.

#### 4. Concluding Remarks

A vortex model is introduced to analyze the spatial evolution of parallel plane jets. In the present study, the following results were obtained:

- (1) In the comparison of the present computational results and previous experimental ones, not only could qualitative spatial behaviors of parallel plane jets be faithfully simulated, but also good quantitative consistence could be obtained.
- (2) In the present results, a highly turbulent production rate exists in the upstreaming neighboring the merging point, and makes an area which is with large stream-wise and transverse turbulence intensities in the downstream neighboring of merging point.
- (3) The values of  $u'_c$  and  $v'_c$  at far-downstream are almost the same for different S/W. The values from parallel plane jets are equal to those from a similar region of a single turbulent plane jet obtained from experiments. The results indicate that at far-downstream, the two jets resemble a turbulent single jet with similar properties.
- (4) In the spectrums of stream-wise fluctuant velocity at the neighbors of combined points for S/W=2 and 5/3m, a clear dominant frequency was found. The increase in reduced frequency accompanies with increasing S/W.

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