飽和多孔性介質內熱輻射與均勻噴吸流效應對混合對流流經等溫垂直圓柱體熱傳遞之影響：整個範圍

Radiation and Blowing/suction Effects on Mixed Convection over an Isothermal Vertical Cylinder in Porous Media: the Entire Regime

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摘要

本文使用數值方法以探討在飽和多孔性介質內，熱輻射與噴吸流效應對混合對流流經等溫垂直圓柱體熱傳遞之影響。先使用變數轉換再以凱勒盒子法解此非相似方程式。數值結果主要顯示無因次溫度分佈和局部 Nusselt 數。當混合對流參數 χ 從 0（自然對流）變化至 1（強制對流）時，局部 Nusselt 數有最低點現象。局部 Nusselt 數因吸入（噴出）流效應而增加（減少）。增加曲率參數 ξ、輻射－傳導參數 R_d 與表面溫度參數 H，則增強局部 Nusselt 數。
關鍵字：輻射，噴吸流，混合對流

Abstract

In this paper numerical solutions are presented for the effects of radiation and blowing/suction on mixed convection about an isothermal vertical cylinder embedded in a saturated porous medium. These partial differential equations are transformed into the nonsimilar boundary layer equations which are solved by an implicit finite-difference method (Keller box method). Numerical results for the dimensionless temperature profiles and the local Nusselt number are presented for the combined convection parameter χ, the transverse curvature parameter ξ, radiation-conduction parameter R_d, surface temperature parameter H and blowing/suction parameter Γ. The entire regime of the mixed convection is included, as χ varies from 0 (pure free convection) to 1 (pure forced convection). As mixed convection parameter χ varies from 0 to 1, the local Nusselt number decreases initially, reaches a minimum in the intermediate value of χ and then increases gradually. The local Nusselt number increases for the effect of suction. On the contrary, the effect of blowing will decrease the local Nusselt number. As the transverse curvature parameter ξ, the radiation-conduction parameter R_d and the surface temperature parameter H increase, the local Nusselt number also increases.
Key Words: Radiation, blowing/suction, mixed convection

1. Introduction

The heat transfer in a porous medium has many important applications in geothermal and geophysical engineering. These include extraction of geothermal energy, thermal insulation of buildings, filtration processes and disposal of underground nuclear wastes. The problems of free convection about a vertical cylinder were studied by Minkowycz and Cheng [1] (VWT), Yücel (UWT and UHF) [2], Kumari et al. (UWT) [3], Huang and Chen (UWT and blowing/suction) [4], Merkin (UWT) [5] and Bassom and Rees (VWT) [6] (the extension of [1, 3, 5]). Forced convection along a longitudinal cylinder embedded in a saturated porous medium was investigated by Vasantha et al. [7]. Darcian mixed convection along a vertical cylinder embedded in a saturated porous medium was analyzed by Merkin and Pop [8] (UWT), Hooper et al. [9] (UWT and the entire regime) and Pop and Na [10] (VHF). Previous researches [1-10], however, have only concentrated upon the problem with no radiation effect.

As the difference between the surface temperature and the ambient temperature is large, it may cause the radiation effect to become important. In the aspect of convection-radiation in porous media, Hossain and Pop [11] analyzed the radiation effect on free convection flow along an inclined surface placed in porous media. Later, Raptis [12] investigated the radiation and free convection flow through a porous medium bounded by a vertical infinite porous plate. Recently, Yih [13-15] studied the radiation effect on free convection over an impermeable vertical cylinder [13], an isothermal vertical permeable flat plate [14], and isothermal vertical permeable cone [15] embedded in porous media. In above researches [11-15], the Rosseland diffusion approximation is employed. This approximation leads to a considerable simplification in the expression for radiant flux.

In the present analysis, we extend the previous work of Hooper et al. [9] and Yih [13] to investigate the effects of radiation and blowing/suction on the heat transfer characteristics in mixed convection over an isothermal vertical cylinder embedded in porous media by using the implicit finite difference method together with Keller box elimination technique.

2. Analysis

Consider the problem of the radiation and blowing/suction effects on mixed convection boundary-layer flow of optically dense viscous incompressible fluids over an isothermal vertical cylinder embedded in a saturated porous medium. The wall temperature of the vertical cylinder T_w is higher than the ambient temperature T_∞ . The x coordinate is measured along the cylinder from the leading edge and the r coordinate is measured from the centerline of the cylinder. The gravitational acceleration g is acting downward along the cylinder, opposite to the x-direction. The uniform free stream U_∞ is moving along the positive x-direction. The surface blowing/suction velocity V_w is also uniform. The variations of fluid properties are limited to density variations which affect the buoyancy force term only. The viscous dissipation effect is neglected for the low velocity. The radiative heat flux in the x-direction is negligible in comparison with that in r-direction. Also, the radius of the cylinder r_o is taken to be much larger than the pore size of the medium so that the flow channeling effect near the cylinder can be neglected.

The governing equations for the problem under consideration with the boundary layer, Boussinesq and Rosseland diffusion approximations [13] and the Darcy law can be written as

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial r}=0, \quad (1)$$

$$\frac{\partial u}{\partial r}=\frac{g\beta K}{\nu}\frac{\partial T}{\partial r}, \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{16\sigma}{3(a_r+\sigma_s)\rho C_p}\frac{1}{r}\frac{\partial}{\partial r}\left(rT^3\frac{\partial T}{\partial r}\right). \quad (3)$$

The boundary conditions are defined as follows:

$$r=r_o: v=V_w, \qquad T=T_w, \quad (4)$$

$$r\rightarrow\infty: u=U_\infty, \qquad T=T_\infty, \quad (5)$$

where u and v are the Darcian velocities in the x and r directions, respectively. β is the thermal expansion coefficient of the fluid. K is the permeability of the porous medium. ν is the kinematic viscosity of the fluid. T is the temperature of the fluid and the porous medium which are in local thermal equilibrium. α is the equivalent thermal diffusivity. σ is the Stefan-Boltzmann constant. a_r is the Rosseland mean extinction coefficient. σ_s is the scattering coefficient. ρ is the density of the fluid. C_p is the specific heat at constant pressure. The term $16\sigma T^3/[3(a_r+\sigma_s)]$ can be considered as the "radiative conductivity".

We now define a stream function ψ such that

$$ru=\partial\psi/\partial r \quad\text{and}\quad rv=-\partial\psi/\partial x. \quad (6)$$

The continuity Eq. (1) is then automatically satisfied. Invoking the following dimensionless variables:

$$\xi=\frac{2x}{r_o(Pe_x^{1/2}+Ra_x^{1/2})} \quad (7a)$$

$$\eta=\frac{r_o(Pe_x^{1/2}+Ra_x^{1/2})}{2x}\left(\frac{r^2}{r_o^2}-1\right) \quad (7b)$$

$$f(\xi,\eta)=\frac{\psi}{\alpha r_o(Pe_x^{1/2}+Ra_x^{1/2})} \quad (7c)$$

$$\theta(\xi,\eta)=\frac{T-T_\infty}{T_w-T_\infty} \quad (7d)$$

and substituting Eq. (7) into Eqs. (1)-(5), we obtain the following transformed governing equations

$$f'=(1-\chi)^2\theta+\chi^2, \quad (8)$$

$$(1+\xi\eta)\theta''+\left(\xi+\frac{1}{2}f\right)\theta'+\frac{4R_d}{3}\left\{(1+\xi\eta)\theta'[(H-1)\theta+1]^3\right\}'=\frac{\xi}{2}\left(f'\frac{\partial\theta}{\partial\xi}-\theta'\frac{\partial f}{\partial\xi}\right). \quad (9)$$

The boundary conditions are defined as follows:

$$\eta=0: \qquad f=-\Gamma, \qquad \theta=1, \quad (10)$$

$$\eta\rightarrow\infty: \qquad \theta=0. \quad (11)$$

In the above equations, the primes denote the differentiation with respect to η. Equation (8) is obtained by integrating Eq. (2) once with the help of Eq. (5). The combined convection parameter χ is defined as

$$\chi=\left(1+\frac{Ra_x^{1/2}}{Pe_x^{1/2}}\right)^{-1}=\text{constant}. \quad (12)$$

It is noted that χ = 0 and 1 correspond to pure free and pure forced convection cases, respectively. The entire regime of mixed convection corresponds to values of χ between 0 and 1. Ra_x and Pe_x are the modified local Rayleigh number for the flow through the porous medium and the local Peclet number defined as

$$Ra_x = \frac{g\beta(T_w - T_\infty)Kx}{\nu\alpha}, \qquad Pe_x = \frac{U_\infty x}{\alpha}. \quad (13)$$

ξ is the transverse curvature parameter. $R_d = 4\sigma T_\infty^3/[k(a_r + \sigma_s)]$ is the radiation-conduction parameter. $H = T_w / T_\infty$ is the surface temperature parameter. Γ is the blowing/suction parameter, defined as

$$\Gamma = \xi\frac{V_w r_o}{2\alpha} = \frac{V_w x}{\alpha(Pe_x^{1/2} + Ra_x^{1/2})}. \quad (14)$$

From Eq. (14), Γ is independent of r_o and can, hence, be employed as a surface blowing/suction parameter [4, 16]. For the case of blowing, V_w > 0 and hence Γ > 0. On the other hand, for the case of suction, V_w < 0 and hence Γ < 0.

In terms of the new variables, the velocity components are given by

$$u = \frac{\alpha(Pe_x^{1/2} + Ra_x^{1/2})^2}{x} f', \quad (15)$$

and

$$v = -\frac{\alpha(Pe_x^{1/2} + Ra_x^{1/2})}{2x}\frac{1}{(1+\xi\eta)^{1/2}}\left(f + \xi\frac{\partial f}{\partial \xi} - \eta f'\right). \quad (16)$$

The heat flux q_w at the surface of the vertical cylinder is

$$q_w = -\left\{\left[k + \frac{16\sigma T^3}{3(a_r + \sigma_s)}\right]\frac{\partial T}{\partial r}\right\}_{r=r_o}. \quad (17)$$

For practical applications, it is usually the local Nusselt number that is of interest. This can be expressed as

$$Nu_x = \frac{hx}{k} = \frac{q_w x}{k(T_w - T_\infty)} \quad (18)$$

where h denotes the local heat transfer coefficient and k represents the thermal conductivity. Substituting Eqs. (7b) and (17) into Eq. (18), we obtain

$$\frac{Nu_x}{(Pe_x^{1/2} + Ra_x^{1/2})} = -\left(1 + \frac{4R_d H^3}{3}\right)\theta'(\xi,0). \quad (19)$$

3. Results and discussion

The above governing Eqs. (8)-(9) and the boundary conditions (10)-(11) are nonlinear partial differential equations depending on the combined convection parameter χ, the transverse curvature parameter ξ, radiation-conduction parameter R_d, surface temperature parameter H and blowing/suction parameter Γ. The present analysis integrates the system of Eqs. (8)-(9) by the implicit finite difference approximation together with the modified Keller box method of Cebeci and Bradshaw [17]. The computations were carried out on a personal computer with $\Delta\xi$ = 0.1 (uniform grid), the first step size $\Delta\eta_1$ = 0.01, the variable grid parameter is chosen 1.01. The value of η_∞ = 1000 was found to be sufficiently accurate for $|\theta_\infty| < 10^{-3}$. The requirement that the variation of the temperature distribution is less than 10^{-5} between any two successive iterations is employed as the criterion of convergence.

In order to verify the accuracy of the present method, we have compared our results with those of Vasantha et al. [7] and Hooper et al. [9]. Table 1 shows the values of $-\theta'(\xi,0)$ with χ = 1, R_d = Γ = 0. Table 2 compares the values of $-\theta'(\xi,0)$ for various values of χ with R_d = Γ = 0. The comparisons in all the above cases are found to be in good agreement. Numerical results are presented graphically for the combined convection parameter χ ranging from 0 to 1, the transverse curvature parameter ξ ranging from 0 to 10, the radiation-conduction parameter R_d ranging from 0 to 10, the surface temperature parameter H ranging from 1.1 to 3.0 and the blowing/suction parameter Γ ranging from -1 to 1.

Table 1. Comparison of values of $-\theta'(\xi,0)$ with χ = 1, R_d = Γ = 0.

ξ	Vasantha et al. [7]	Hooper et al. [9]	Present results
0.0	0.5644	0.5642	0.5642
1.0	0.7889	—	0.7890
2.0	0.9852	0.9841	0.9837
3.0	1.1702	—	1.1618
4.0	1.3499	1.3293	1.3287
5.0	1.5263	—	1.4874
6.0	—	—	1.6398
7.0	—	—	1.7871
8.0	—	—	1.9301
9.0	—	—	2.0694
10.0	2.3624	2.2078	2.2057

Table 2. Comparison of values of $-\theta'(\xi,0)$ for various values of χ with R_d = Γ = 0.

χ	Hooper et al. [9]		Present results	
	ξ=2	ξ=10	ξ=2	ξ=10
0.1	0.7386	1.7494	0.7342	1.7279
0.2	0.7113	1.7250	0.7088	1.7066
0.3	0.7032	1.7360	0.7006	1.7161
0.4	0.7111	1.7664	0.7095	1.7522
0.5	0.7347	1.8193	0.7336	1.8082
0.6	0.7708	1.8857	0.7699	1.8773
0.7	0.8159	1.9599	0.8152	1.9544
0.8	0.8679	2.0402	0.8671	2.0361
0.9	0.9243	2.1232	0.9237	2.1203

Figure 1 shows the dimensionless temperature profiles for various values of ξ with Γ = -0.5, R_d = 10, H = 1.1, χ = 0.5. Increasing the transverse curvature parameter ξ increases the dimensionless surface temperature gradient. The dimensionless temperature profiles for various values of Γ with ξ = 1, R_d = 5, H = 2.0, χ = 0.5 is shown in Fig. 2. The effect of suction decreases the thermal boundary layer. Whereas, for the case of blowing increases the thermal boundary layer. Figure 3 illustrates the dimensionless temperature profiles for various values of H with ξ = 5, Γ = -1, R_d = 1, χ = 0.5. It is observed that the dimensionless temperature profiles increase due to the increase in the surface temperature parameter H. The dimensionless temperature profiles for various values of R_d with ξ = 2, Γ = 1, H = 1.5, χ = 0.5 is displayed in Fig. 4. Increasing the radiation-conduction parameter R_d enhances the dimensionless temperature profiles. As the radiation-conduction parameter R_d and the surface temperature parameter H are changed, the dimensionless temperature profiles tend to be similar to Figs. 3-4.

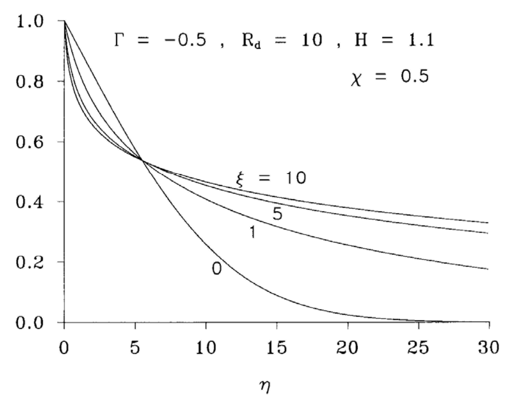

Fig. 1. Dimensionless temperature profiles for various values of ξ

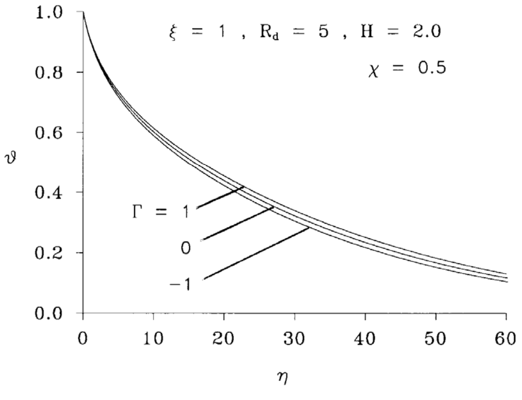

Fig. 2. Dimensionless temperature profiles for various values of Γ

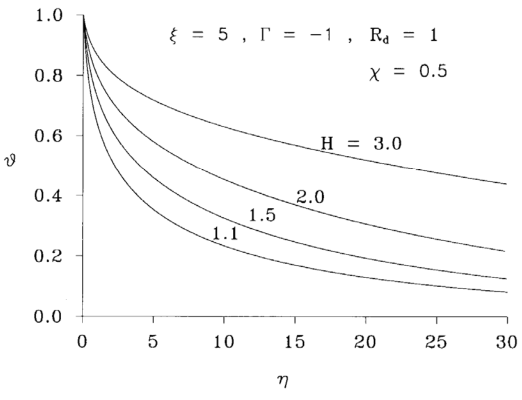

Fig. 3. Dimensionless temperature profiles for various values of H

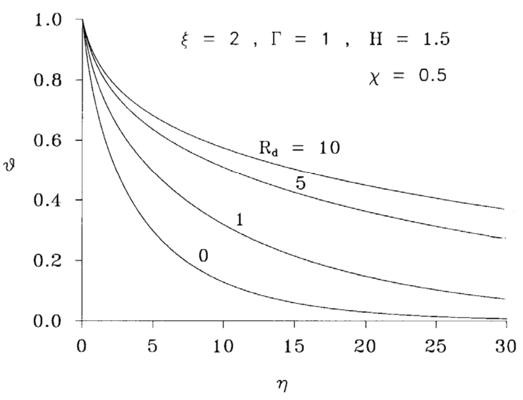

Fig. 4. Dimensionless temperature profiles for various values of R_d

Figure 5 shows the local Nusselt number for various values of the transverse curvature parameter ξ with Γ = -0.5, R_d = 10, H = 1.1. It is observed that the local Nusselt number increases as the transverse curvature parameter increases, at a given χ . This is because that increasing the transverse curvature parameter ξ increases the dimensionless temperature gradient at the wall (Fig. 1). The local Nusselt number for various values of blowing/suction parameter Γ with ξ = 1, R_d = 5, H = 2.0 is displayed in Fig. 6. The local Nusselt number is increasing for the effect of suction, i.e. $\Gamma < 0$, for a fixed χ . The opposite is true for the effect of blowing. This is because that suction (blowing) case decreases (increases) the thermal boundary layer (Fig. 2).

Figure 7 illustrates the local Nusselt number for various values of the surface temperature parameter H with ξ = 5, Γ = -1, R_d = 1. Increasing the surface temperature parameter H enhances the local Nusselt number. The local Nusselt number for various values of radiation-conduction parameter R_d with ξ = 2, Γ = 1, H = 1.5 is shown in Fig. 8. It is seen that the value of the local Nusselt number increases with increasing the value of R_d.

Moreover, as χ varies from 0 to 1, the local Nusselt number decreases initially, reaches a minimum at an intermediate value of χ and then increases gradually, as shown in Figs. 5-8. The phenomenon of minimum is also found in Hooper et al. [9] for the case of $R_d = \Gamma$ = 0. This minimum does not imply a corresponding minimum value in the local Nusselt number. This is due to the nature of $Nu_x/(Pe_x^{1/2}+Ra_x^{1/2})$ vs. χ plot. For example, let us consider the present results of $\xi = 2$, $\Gamma = 1$, $H = 1.5$, $R_d = 1$ and $\chi = 0.5$ in Fig. 8. If the local Peclet number is taken as $Pe_x = 100$, the corresponding modified local Rayleigh number can be found to be $Ra_x = 100$ from Eq. (12). Using the present numerical results of $Nu_x/(Pe_x^{1/2} + Ra_x^{1/2})$, the value of Nu_x for mixed convection ($Pe_x = 100$, $Ra_x = 100$) is 36.226. While, for pure free convection ($\chi = 0.0$) and pure forced convection ($\chi = 1.0$) the Nu_x values are found to be 19.228 and 23.697, respectively. Therefore, it is obvious that the present result of Nu_x for mixed convection is higher than that for pure free convection and pure forced convection.

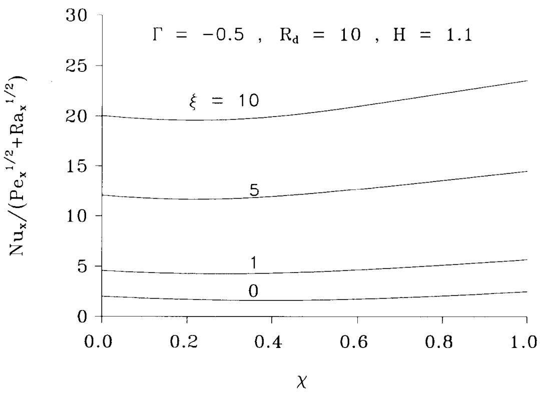

Fig. 5. Local Nusselt number for various values of ξ

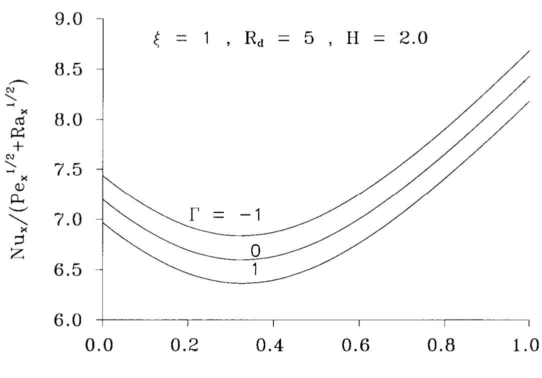

Fig. 6. Local Nusselt number for various values of Γ

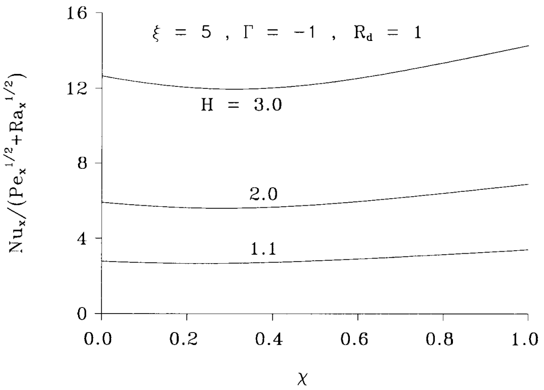

Fig. 7. Local Nusselt number for various values of H

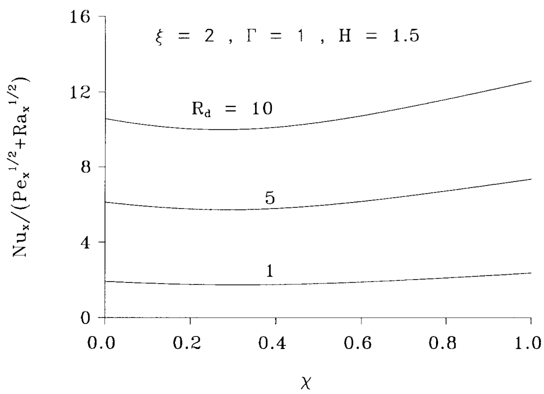

Fig. 8. Local Nusselt number for various values of R_d

4. Conclusions

The radiation and blowing/suction effects on the mixed convection flow of an optically dense viscous fluid adjacent to an isothermal vertical cylinder embedded in a saturated porous medium with Rosseland diffusion approximation is numerically investigated. The transformed nonsimilar conservation equations are obtained and solved by the Keller box method. Numerical results are given for the dimensionless temperature profiles and the local Nusselt number for various values of the combined convection parameter χ, the transverse curvature parameter ξ, the radiation-conduction parameter R_d, the surface temperature parameter H and the blowing/suction parameter Γ. It is apparent that increasing the transverse curvature parameter ξ, the radiation-conduction parameter R_d, the surface temperature parameter H and decreasing the blowing/suction parameter Γ (for the case of suction) will enhance the local Nusselt number. The variation of the local Nusselt number with the increase of χ has the phenomenon of minimum.

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