### Determine the Best Receiver Location for LEO Satellite Communications under Elevation Angle Dependent Channel Model

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#### **ABSTRACT**

A method is proposed to evaluate the BER performance and the visibility probability for LEO satellite communication system where the propagation path varies with time. The BER represents the link quality of the received signal, which is derived by incorporating the probability density function (PDF) of elevation angles and the elevation-angle-dependent channel model. While the visibility probability represents the time duration that the receiver can communicate with satellite, which is obtained by the PDF of maximum elevation angles and the visibility time duration of each satellite path corresponding to the specific maximum elevation angle. MPSK communication scheme is tested to show that both BER performance and visibility probability depend on the latitude of the receiver as well as satellite orbit parameters such as the altitude and the inclination angle. The proposed method would be helpful for system designers to determine the receiver location or the satellite orbit parameters in order to obtain the best BER or the visibility probability is the largest.

**Keywords:** BER, MPSK, LEO, elevation angle, visibility

### 在仰角相依通道下決定低軌衛星通信的最佳接收機位置

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### 摘 要

本文對低軌道衛星具時變性通道路徑提出位元錯誤率與可視機率計算方法。位元錯誤率代表接收信號品質,由仰角機率密度函數結合仰角相依通道推得。可視機率則表示接收機與衛星通信時期,可由最大仰角機率密度函數推得,而每顆衛星可視時期等於特定的最大仰角。以MPSK 模擬證明位元錯誤率、可視機率是相依於接收站緯度位置與衛星軌道參數。本文的方法將有助於衛星星系系統設計者決定接收站建置最佳位置及獲得衛星軌道參數。

關鍵詞:位元錯誤率、多位元相位位移鍵、低軌道衛星、仰角、可視機率

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### I. INTRODUCTION

LEO satellite communication provides an essential way of data transmission for such diverse purpose as weather forecasting, remote sensing, Internet communication, navigation and personal communication systems [1]. To keep good link quality, it is essential to understand the BER performance under different conditions in the designing stage. A lot of channel model is proposed in the literature to describe the PDF of the received signal at a specific condition. For example, in [2]-[4], the proposed channel models consider the line-of-sight (LOS) component under shadowing is log-normally distributed, and the multipath effect is Rayleigh distributed. These channel models are suitable orbit (GEO) satellite geo-stationary communication system or the condition that the elevation angle of propagation path does not change the channel fading effect. In [5] and [6], an elevation-angle-dependent channel models for non-GEO satellite have been proposed to describe the fading statistics of the received signal but only at some specific elevation angles. However, as satellite is constantly moving in and out of view, the elevation angle of the propagation path also varies with time for a given ground location. As a result, the probability of elevation angles should be taken into account to obtain the statistical properties of the received signal.

Additionally, if the visibility time duration is concerned, such as remote sensing satellite, the satellite orbit parameters and the ground receiver location should be carefully designed in order to obtain the maximum communication time duration in one day. Unfortunately, up to now, performing an orbital simulation seems to be the only way to obtain the visibility time duration for a specific ground receiver [7][8], which requires substantial numerical efforts to process great deal of path data for each specific receiver location under consideration. Therefore, a simplified method is need.

In this paper, we develop a method to quickly and exactly obtain the BER performance and the visibility probability for a given ground receiver. As the derivation bases on geometry analysis, orbital simulations are not necessary.

### II. GEOMETRY ANALYSIS

#### 2.1 The PDF of Subsatellite Points

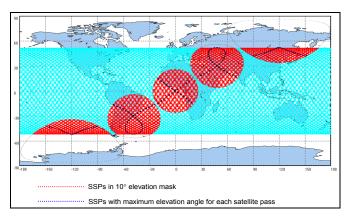


Fig. 1. The ground tracks of a 48 Globalstar satellites.

Fig.1. shows the ground tracks of a 48 Globalstar satellites in 0.5-day orbit duration. From the figure we see that, due to the Earth's rotation, the probability for satellite stays at higher latitude is larger than that of stays at lower latitude. Since the latitude-dependent distribution of subsatellite points (SSPs) influence the statistics of satellite links in the elevation mask of a ground receiver, it is desired to develop an analytical model to express the distribution of SSPs on the Earth's surface.

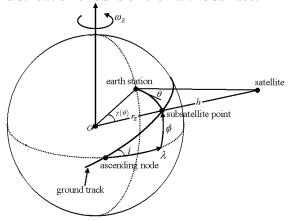


Fig. 2. Earth-satellite geometry.

Let a satellite orbit the Earth with inclination i and radius  $r_s = r_E + h$ , where

 $r_E = 6378.145$  km is the radius of the Earth and h is the orbit altitude. Fig.2. shows the Earth-satellite geometry where the Earth centered fixed (ECF) coordinate system is employed. Although in the ECF frame the satellite's orbit is not a great circle due to the rotation of the earth, the authors in [9] shows that the LEO satellite's orbit during the in-view period can be approximated to be a great-circle since the time duration of a satellite passing through the elevation mask is small compared to the orbit period. Based on this assumption, the ground track can be obtained by using the law of spherical triangles, given by [10]

$$\phi = \sin^{-1} \left( \sin i \cdot \sin \lambda \right) \tag{1}$$

where  $\phi$  and  $\lambda$ , measured from the ascending node, are the latitude and longitude of the SSP in the ground track, respectively. Since the angular velocity of the satellite can be approximated by a constant [9],  $\lambda$  can be regarded as uniformly distributed in  $[0,2\pi)$ , i.e.,

$$f_{\Lambda}(\lambda) = \frac{1}{2\pi}, \quad 0 \le \lambda < 2\pi$$
 (2)

To obtain the PDF of  $\phi$ 's, the random variable transformation [11, p.86] is employed on (1) and (2). As a result, the PDF of SSPs, expressed as a function of geographical latitude  $\phi$ , is obtained by

$$f_{\Phi}(\phi) = \begin{cases} \frac{\cos\phi}{\pi\sqrt{\sin^2 i - \sin^2\phi}}, & |\phi| < i\\ 0 & \text{elsewhere} \end{cases}$$

## 2.2 Statistical Properties in a Single Satellite Pass

In this section the visibility time duration and the PDF of elevation angles in a single satellite pass is derived. For a given satellite pass, the visibility time duration is shown to be closely related to the maximum elevation angle  $\theta_{\max}$  and can be approximated by [9]

$$\tau(\theta_{\text{max}}) \approx \frac{2}{\omega_{\text{S}} - \omega_{\text{E}} \cos i} \cdot \cos^{-1} \left[ \frac{\cos \gamma(\theta_{\text{min}})}{\cos \gamma(\theta_{\text{max}})} \right]$$
 (4)

where  $\omega_{S}$  and  $\omega_{E}$  are the angular velocities of the satellite and the earth, respectively.  $\theta_{\min}$  is the predetermined minimum elevation angle, and  $\gamma(\theta)$  is the central angle between the earth station and the SSP corresponding to elevation angle  $\theta$ . In Fig.2., the relationship between  $\gamma$  and  $\theta$  follows

$$\gamma(\theta) = \cos^{-1}(a \cdot \cos \theta) - \theta \tag{5}$$

where  $a = r_E/r_S$  and  $r_E$  and  $r_S$  represent the radius of the Earth and satellite orbit, respectively. To be specific, Fig.3. shows the changes of elevation angle as s function of time at different maximum elevation angle, taking the orbit altitude h = 1414 km as an example. It is worth noting that the time duration of each satellite path depends only on the orbit altitude and, the higher the elevation angle, the lower the occurrence probability. From (4) we see that, provided the  $\theta_{max}$  is known, the time interval of the satellite's appearance above the specific  $\theta$  can be obtained by

$$d(\theta) = \frac{2}{\omega_{S} - \omega_{E} \cos i} \cos^{-1} \left[ \frac{\cos \gamma(\theta)}{\cos \gamma(\theta_{\text{max}})} \right]$$

$$, \ \theta_{\text{min}} \le \theta \le \theta_{\text{max}}$$
(6)

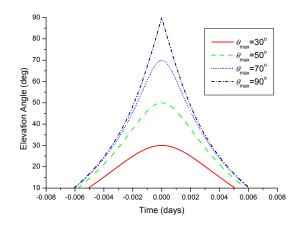


Fig. 3. The changes of elevation angle as a function of time at different maximum elevation angle.

Note that  $d(\theta_{\min}) = \tau(\theta_{\max})$ . The cumulated distribution function (CDF) of elevation angles for this single satellite pass can be expressed as

$$F_{S}(\theta) = 1 - \frac{d(\theta)}{d(\theta_{\min})}, \ \theta_{\min} \le \theta \le \theta_{\max}$$
 (7)

and the corresponding PDF is given by

$$f_{S}(\theta) = \frac{dF_{S}(\theta)}{d\theta}$$

$$= \frac{G(\theta)\sin\gamma(\theta)}{\sqrt{\cos^{2}\gamma(\theta_{\text{max}}) - \cos^{2}\gamma(\theta)} \cdot \cos^{-1}\left(\frac{\cos\gamma(\theta_{\text{min}})}{\cos\gamma(\theta_{\text{max}})}\right)}$$

$$,\theta_{\text{min}} \leq \theta$$
(8)

where

$$G(\theta) = \frac{1 + a^2 - 2a\cos\gamma(\theta)}{1 - a\cos\gamma(\theta)} \tag{9}$$

# 2.3 The PDF of Maximum Elevation Angles

As shown in (4) and (8), the maximum elevation of a given satellite pass,  $\theta_{\rm max}$ , determines the visibility time duration and the distribution of elevation angles from rise to set, respectively. As a result, for a given earth station, the occurrence probability of any specific satellite pass can be specified by the PDF of  $\theta_{\rm max}$  's, denoted as  $f_{\Theta_{\rm max}}(\theta_{\rm max})$ . To find  $f_{\Theta_{\rm max}}(\theta_{\rm max})$ , let us consider the elevation mask consisting of a number of satellite passes. The SSP corresponding to  $\theta_{\rm max}$  in a given satellite pass has the smallest central angle  $\gamma_{\rm min}$  to the earth station, where  $\gamma_{\rm min} = \gamma(\theta_{\rm max})$ .

Our extensive simulation results show that for an earth station located at latitude  $\phi_0$ , the probability of  $\gamma_{\min}$ 's, denoted as  $f_{\Gamma_{\min}}\left(\gamma_{\min}\right)$ , to a high degree of accuracy, can be approximated by  $f_{\Phi}\left(\phi_0-\gamma_{\min}\right)+f_{\Phi}\left(\phi_0+\gamma_{\min}\right)$ . Note that  $f_{\Phi}\left(\phi_0+\gamma_{\min}\right)=0$  if  $\phi_0+\gamma_{\min}\geq i$ . As a result, the PDF of  $\gamma_{\min}$ 's will be

$$f_{\Gamma_{\min}}(\gamma_{\min})$$

$$= \frac{1}{K_{\perp}} \left[ f_{\Phi}(\phi_0 + \gamma_{\min}) + f_{\Phi}(\phi_0 - \gamma_{\min}) \right]$$
(10)

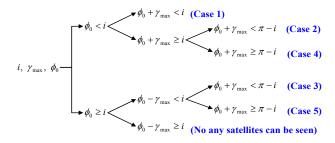
where

$$\begin{split} K_{\phi_0} &= \int\limits_{-\gamma_{\text{max}}}^{\gamma_{\text{max}}} f_{\Phi} \left( \phi_0 + \gamma_{\text{min}} \right) d\gamma_{\text{min}} \\ &= \frac{1}{\pi} \left[ \sin^{-1} \left( \frac{\sin \left( \phi_0 + \gamma \left( \theta_M \right) \right)}{\sin i} - \frac{\sin \left( \phi_0 - \gamma_{\text{max}} \right)}{\sin i} \right) \right] \end{split} \tag{11}$$

is the normalization constant,  $\gamma_{\rm max}$  is the central angle corresponding to  $\theta_{\rm min}$ . i.e.,  $\gamma_{\rm max} = \gamma \left(\theta_{\rm min}\right)$ . The PDF in (10), from our simulation results, is expressed in terms of central angle. By applying random variable transformation on (10), according to (5), the PDF of  $\theta_{\rm max}$ 's is obtained by

$$f_{\Theta_{\text{max}}}(\theta_{\text{max}}) = \frac{G(\theta_{\text{max}})}{K_{\phi_{\bullet}}} \cdot \left[ f_{\Phi}(\phi_{0} + \gamma(\theta_{\text{max}})) + f_{\Phi}(\phi_{0} - \gamma(\theta_{\text{max}})) \right]$$
(12)

Since  $f_{\Phi}(\phi_0 + \gamma_{\min}) = 0$  under the condition of  $\phi_0 + \gamma_{\min} \ge i$ , to be specific, five cases are classified according to the latitude of the earth station, as shown in Fig.4.



F.ig.4. Classification of types of elevation angle PDF according to earth station's latitude.

Case 1)  $0 \le \phi_0 < i - \gamma_{\text{max}}$ : As shown in Fig. 1., this case implies that the earth station's location and its elevation mask are in the latitude of  $\pm i$ , i.e., the elevation mask is filled with SSPs. The PDF of maximum elevation angles is expressed as

$$\begin{split} &f_{\Theta_{\text{max}}}\left(\theta_{\text{max}}\right) \\ &= \frac{G\left(\theta_{\text{max}}\right)}{K_{1}} \Big[ f_{\Phi}\left(\phi_{0} + \gamma\left(\theta_{\text{max}}\right)\right) + f_{\Phi}\left(\phi_{0} - \gamma\left(\theta_{\text{max}}\right)\right) \Big] (13) \\ &, \theta_{\text{min}} \leq \theta_{\text{max}} < \frac{\pi}{2} \end{split}$$

Case 2)  $i - \gamma_{\text{max}} \le \phi_0 < i$ : This case implies that the earth station is located within latitude  $\pm i$  while part of its elevation mask go beyond the latitude  $\pm i$ , i.e., only part of the elevation mask is filled with SSPs. In that case,

$$f_{\Theta_{\max}}(\theta_{\max}) = \begin{cases} \frac{G(\theta_{\max})}{K_2} \cdot f_{\Phi}(\phi_0 - \gamma(\theta_{\max})), & \theta_{\min} \leq \theta_{\max} < \theta_C \\ \frac{G(\theta_{\max})}{K_2} \Big[ f_{\Phi}(\phi_0 + \gamma(\theta_{\max})) + f_{\Phi}(\phi_0 - \gamma(\theta_{\max})) \Big], & \theta_C < \theta_{\max} < \frac{\pi}{2} \end{cases}$$

$$(14)$$

where

$$\theta_C = \tan^{-1} \left( \frac{\cos(i - \phi_0) - a}{\sin(i - \phi_0)} \right)$$
 (15)

denotes the corresponding elevation angle when the SSP locates at latitude i.

Case 3)  $i \le \phi_0 < \pi - i - \gamma_{\max}$ : The earth station is located beyond the scope of satellite track, i.e., the latitude of  $\pm i$ . However, still part of the elevation mask is within the latitude  $\pm i$ . As a result,

$$f_{\Theta_{\text{max}}}\left(\theta_{\text{max}}\right) = \frac{G\left(\theta_{\text{max}}\right)}{K_{3}} \cdot f_{\Phi}\left(\phi_{0} - \gamma\left(\theta_{\text{max}}\right)\right),$$

$$\theta_{\text{min}} \leq \theta_{\text{max}} < \tan^{-1}\left(\frac{\cos\left(\phi_{0} - i\right) - a}{\sin\left(\phi_{0} - i\right)}\right)$$
(16)

where  $K_i$ , i = 1..3 is the normalization constant for ith case, we have

$$K_{1} = \frac{1}{\pi} \left( \sin^{-1} \left( \frac{\sin \left( \phi_{0} + \gamma_{\max} \right)}{\sin i} \right) - \sin^{-1} \left( \frac{\sin \left( \phi_{0} - \gamma_{\max} \right)}{\sin i} \right) \right)$$
(17)

$$K_{2} = K_{3} = \int_{-\gamma_{\text{max}}}^{i-\phi_{0}} f_{\Phi}(\phi_{0} + \gamma_{\text{min}}) d\gamma_{\text{min}}$$

$$= \frac{1}{2} - \frac{1}{\pi} \sin^{-1} \left( \frac{\sin(\phi_{0} - \gamma_{\text{max}})}{\sin i} \right)$$
(18)

Case 4)  $\pi - i - \gamma_{\text{max}} \le \phi_0 < i$ : This case implies that the earth station is located within latitude  $\pm i$  while the elevation mask involves the North pole and then reaches the opposite sweep region of the satellite track.

Case 5)  $\pi - i - \gamma_{\text{max}} \le \phi_0$  and  $i < \phi_0$ : This case implies that the earth station is located beyond the latitude  $\pm i$  while the elevation mask involves the north pole and then reaches the opposite sweep region of the satellite track.

### 2.4 The PDF of Elevation Angles

In (12) we have derived the PDF of the maximum elevation angles for a single satellite pass. However, what we are interested is the PDF of elevation angles considering all satellite passes in the visibility region. Let us consider the total visibility time duration  $T_{\nu}$ , which is the sum of time durations over all satellite passes, we have

$$T_{v} = \int_{\theta_{\min}}^{\theta_{M}} \tau(\theta_{\max}) f_{\Theta_{\max}}(\theta_{\max}) d\theta_{\max}$$
 (19)

where  $\theta_M$  is specified as the maximum elevation angle that can be observed overall satellite passes. For Case 1 and Case 2,  $\theta_M = \pi/2$  since the earth station has the opportunity to observe satellite at an elevation angle of 90°; while in Case 3 the earth station locates beyond latitude i,  $\theta_M$  is obtained by

$$\theta_{M} = \tan^{-1} \left( \frac{\cos(\phi_{0} - i) - a}{\sin(\phi_{0} - i)} \right) \tag{20}$$

The conditional PDF of elevation angles, given  $\theta_{\rm max}$  , can be obtained by

$$f_{\Theta|\Theta_{\max}}(\theta|\theta_{\max}) = f_{S}(\theta) \frac{\tau(\theta_{\max})}{T_{v}}$$

$$,\theta_{\min} \leq \theta < \theta_{\max}$$
(21)

Finally, the joint PDF of  $\theta$  's and  $\theta_{\rm max}$  's can be interpreted as

$$f_{\Theta,\Theta_{\text{max}}}(\theta,\theta_{\text{max}})$$

$$= f_{\Theta|\Theta_{\text{max}}}(\theta|\theta_{\text{max}})f_{\Theta_{\text{max}}}(\theta_{\text{max}})$$

$$= \frac{G(\theta)\sin\gamma(\theta)}{\sqrt{\cos^{2}\gamma(\theta_{\text{max}}) - \cos^{2}\gamma(\theta)}} \cdot \frac{f_{\Theta_{\text{max}}}(\theta_{\text{max}})}{\int_{\theta_{\text{min}}}^{\theta_{M}} f_{\Theta_{\text{max}}}(x)\cos^{-1}\left(\frac{\cos\gamma(\theta_{\text{min}})}{\cos\gamma(x)}\right) dx}$$

$$, \theta_{\text{min}} \leq \theta < \theta_{\text{max}}$$

$$(22)$$

And the PDF of  $\theta$  's is obtained by integrating (22) over the range of  $\theta_{\text{max}}$ , given by

$$f_{\Theta}(\theta) = \int_{a}^{\theta_{M}} f_{\Theta,\Theta_{\text{max}}}(\theta, \theta_{\text{max}}) d\theta_{\text{max}}$$
 (23)

# III. DETERMINE THE BEST RECEIVER STATION

### 3.1 Methodology

Given the knowledge of satellite orbit altitude h and inclination angle i, and consider the propagation channel model is elevation angle dependent, this section discusses the methodology of determining the best receiver with minimum BER or maximum visibility probability. shown Fig.1., As in 10° elevation mask of each ground receiver consists of many satellite passes with different  $\theta_{max}$ , and each satellite pass is made up of continuous SSPs. To determine the location of ground receiver, all interested receivers located at different latitude is surveyed and the receiver latitude in which the received signal has minimum BER, or the visibility time duration is the largest, comparing to other receiver located

at any latitude, is selected as the best receiver location.

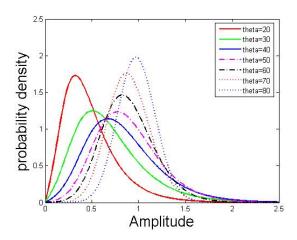


Fig.5. The received signal amplitude PDF for Rice-lognormal channel.

# 3.2 BER Performance under Elevation-angle-dependent Rice lognormal (RLN) Channel Model

In this type channel model, the elevation angle of satellite link  $\theta$  inferences the PDF of the received signal, the parameter K (Rice factor),  $\mu$  (mean),  $\sigma$  (variance) are modeled as function of  $\theta$  and the resulting formulas can refer to reference [6] and the received signal amplitude PDF as Fig.5.

And in [6], Corazza and Vatalaro proposed M-ary PSK with coherent demodulation, the bit error probability in RLN channel derived from [6] as

$$P_{e} = \int_{0}^{\infty} P(e|r)p_{r}(r)dr = E_{s}\left\{E_{R}\left[P(e|r)\right]\right\}$$
 (24)

where P(e|r) is the symbol error probability conditioned on r,  $p_r(r)$  is RLN PDF of received signal envelope r  $E_X\{\bullet\}$  denotes expectation about X. And

$$P_{e}$$

$$= \overline{N}_{a} E_{s} \left\{ Q(U, V) - \frac{1}{2} \left[ 1 + \sqrt{\frac{p}{1+p}} \right] \right\}$$

$$\cdot \exp \left( -\frac{U^{2} + V^{2}}{2} I_{0}(UV) \right)$$
(25)

where Q(U,V) is the Marcum's Q-function, and

$$\frac{U}{V} = \sqrt{K} \left[ \frac{1+2p}{2(1+p)} \mp \sqrt{\frac{p}{1+p}} \right]^{1/2} , \quad \text{where}$$

 $p=p_0/K+1$ ,  $p_0=S^2\rho$ ,  $\rho=d^2/8\sigma_\varepsilon^2$  and  $\sigma_\varepsilon^2$  is the variance of the disturbance acting within the channel,  $\overline{N}_a$  and d are the average number of adjacent points and the minimum Euclidean distance within the signal constellation, respectively.

But for Elevation-angle-dependent Rice lognormal (RLN) Channel Model , the elevation angle of satellite link  $\theta$  is time varying. To take account of the characteristic of elevation angle of satellite link  $\theta$ , the fading amplitude r can be express as a function of elevation angle  $\theta$  and the corresponding occurrence probability for each elevation is obtain as (23), as a result, the average performance (24) should be express as (26). We have

$$P_{e} = E_{\theta} \left\{ E_{s} \left\{ E_{R} \left[ P(e|r) \right] \right\} \right\}$$

$$= E_{\theta} \left\{ \overline{N}_{a} E_{S} \left\{ Q(U,V) - \frac{1}{2} \left[ 1 + \sqrt{\frac{p}{1+p}} \right] \right\} \right\}$$

$$\cdot \exp \left( -\frac{U^{2} + V^{2}}{2} \right) \cdot I_{0}(UV)$$

$$(26)$$

### 3.3 Determine the Visibility Probability

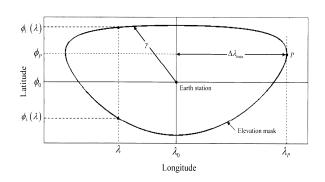


Fig. 6. The geometry of elevation mask.

Consider a circular LEO satellite around the Earth at an inclination of i and an altitude of h. For earth station located at latitude  $\phi_0$  and longitude  $\lambda_0$  with minimum elevation angle  $\theta_{\min}$ . Fig.6. shows the boundary of elevation mask as a function of latitude  $\phi$  and longitude  $\lambda$ , which is a small circle of radius  $\gamma$  as (5).

By spherical geometry [13, p.727], the relationship among  $\gamma$ ,  $\phi$  and  $\lambda$  is

$$\cos \gamma = \sin \phi \sin \phi_0 + \cos \phi \cos \phi_0 \cos (\lambda - \lambda_0)$$
 (27)

If the satellite parameters and the location of the earth station are given, the parameters  $\gamma$ ,  $\phi_0$  and  $\lambda_0$  are fixed. In that case, to express  $\lambda$  as a function of  $\gamma$ ,  $\phi$ ,

$$\lambda = \lambda_0 \pm \cos^{-1} \left( \frac{\cos \gamma - \sin \phi \sin \phi_0}{\cos \phi \cos \phi_0} \right)$$

$$, \quad \phi_0 - \gamma \le \phi \le \phi_0 + \gamma$$
(28)

For a given earth station located at latitude  $\phi_0$ , the ground trace of satellite should be within the visibility perimeter, i.e. the SSPs located in the elevation mask. If the visibility time duration is concerned, the in-view period of each possible satellite pass is the shift time of elevation mask longitude distance  $\Delta\lambda$  [12] and then weighted by the corresponding probability of latitude,  $f_{\Phi}(\phi)$  as (3). We have

$$T = \int_{\phi_{\min}}^{\phi_{\max}} \frac{\Delta \lambda(\phi)}{2\pi} f_{\Phi}(\phi) d\phi$$

$$= \int_{\phi_{\min}}^{\phi_{\max}} \frac{\cos^{-1}\left(\frac{\cos \gamma_{\max} - \sin \phi \sin \phi_{0}}{\cos \phi \cos \phi_{0}}\right) \cdot \cos \phi}{\pi^{2} \sqrt{\sin^{2} i - \sin^{2} \phi}} d\phi$$
(29)

where

$$B(\phi) = \frac{\cos \gamma_{\text{max}} - \sin \phi \sin \phi_0}{\cos \phi \cos \phi_0}$$
 (30)

where  $T_E$  is the Earth self-rotation time and thus the visibility probability is considered as

$$P_{T} = \int_{\phi_{\min}}^{\phi_{\max}} \frac{\cos^{-1} B(\phi) \cdot \cos \phi}{\pi^{2} \sqrt{\sin^{2} i - \sin^{2} \phi}} d\phi$$
 (31)

Since  $\cos^{-1} B(\phi) \in [0,\pi]$ , some modifications should be done as (32), so that (31) is applicable for high latitude  $\phi$  and  $\phi_0$ . Even in [13], the author never make mention of it. We should have

$$\Delta\lambda(\phi) = \begin{cases} \pi - \cos^{-1}(1 + B(\phi)), & -1 \le B(\phi) \le 0\\ \pi, & |B(\phi)| > 1 \\ \cos^{-1}(B(\phi)), & \text{elsewhere} \end{cases}$$
(32)

# IV. SIMULATION AND NUMERICAL RESULTS

We chose Globalstar system, in which all satellites orbit the earth with h = 1414 km and  $i = 52^{\circ}$ , to illustrate the effectiveness of our model. Let  $\theta_{\min} = 10^{\circ}$  we have  $\gamma_{\max} = 0.4587$  rad. By Using BPSK communication scheme as an example, Fig. 7. shows the BER performance for different receiver latitude, taken 5dB, 10 dB, 15dB and 20dB as the tasted received SNR. Theoretical expression matches the numerical simulation results, which are obtained by using standard BPSK signaling and random number generator that generates channel fading effects equivalent to elevation-angle dependent channel model. Note that the BER performance depends

on receiver latitude  $\phi_0$  and  $\phi_0 \approx 45^\circ$  is consider as the best receiver latitude for Globalstar systems.

Fig.8. shows the theory and orbital simulation result of visibility probability observed from ground receiver located at different latitude to Globalstar satellite. Both curves show that receiver located at about  $\phi_0 \approx 40^\circ$  latitude obtains the best visibility probability.

### V. CONCLUSIONS

In this paper we propose methods to determine the ground station having minimum BER performance or maximum visibility probability for LEO satellite communication system, which is latitude dependent and is symmetrical for both the North and the South hemisphere. Based on geometry analysis, the PDF of elevation angles and visibility probability in the elevation mask for ground station located at different latitude are derived. Our method does not need any orbital simulation and path data process, it provides a speedy way to determine the optimal ground station location and satellite orbit parameters for system designer in the system designing stage.

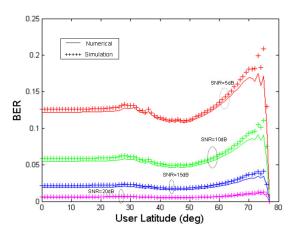


Fig.7. BPSK BER performance at different receiver latitude for Globalstar satellite system.

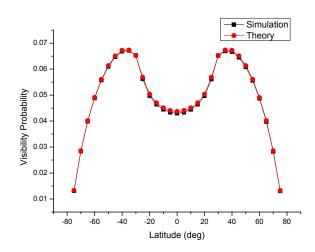


Fig. 8. Visibility probability observed from ground receiver located at different latitude to Globalstar satellite.

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